

# Age of Information in Random Access Channels

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- We design **for the first time decentralized age-based** transmission policies
- Provide analytical results on the age of information

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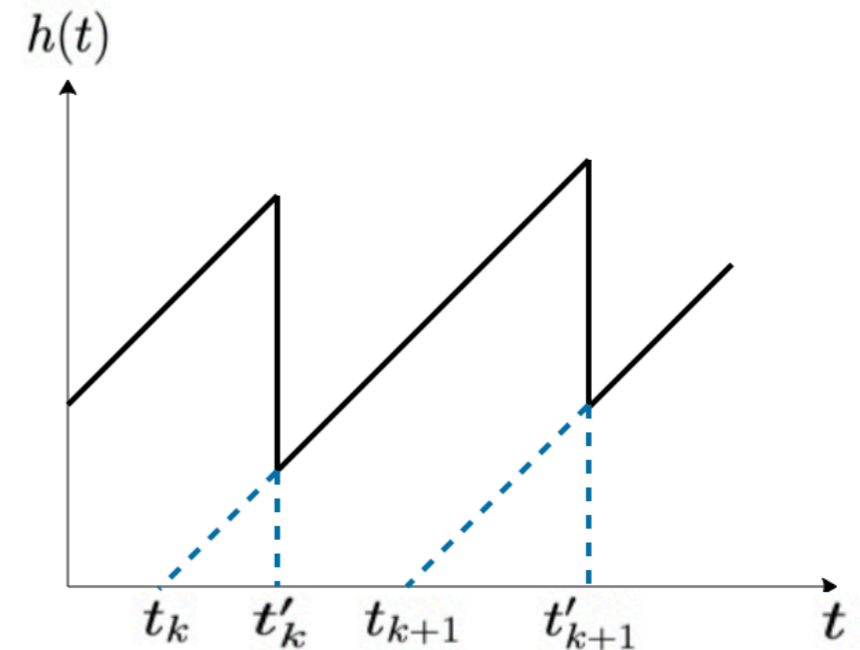


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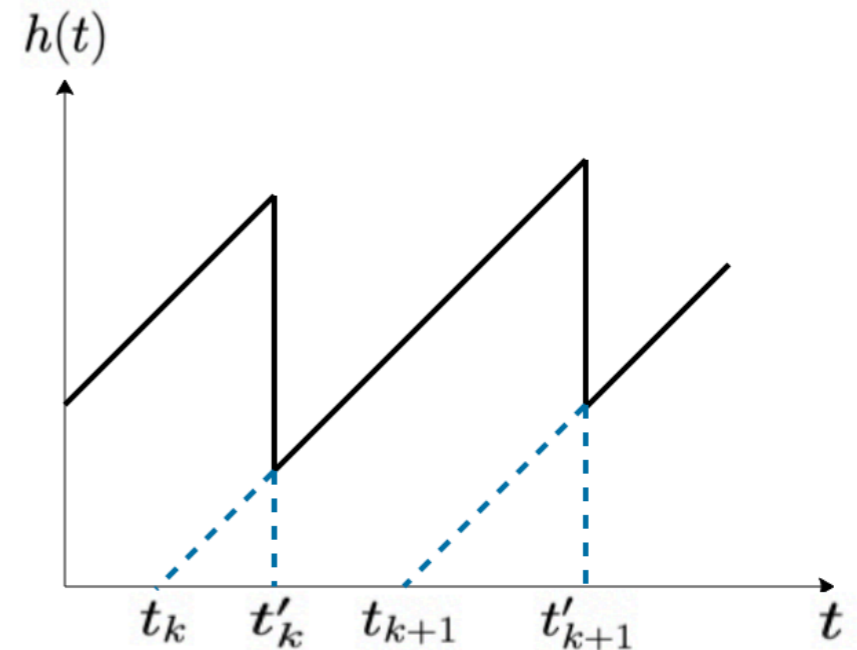
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- Time average age:  $\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T h(t)$



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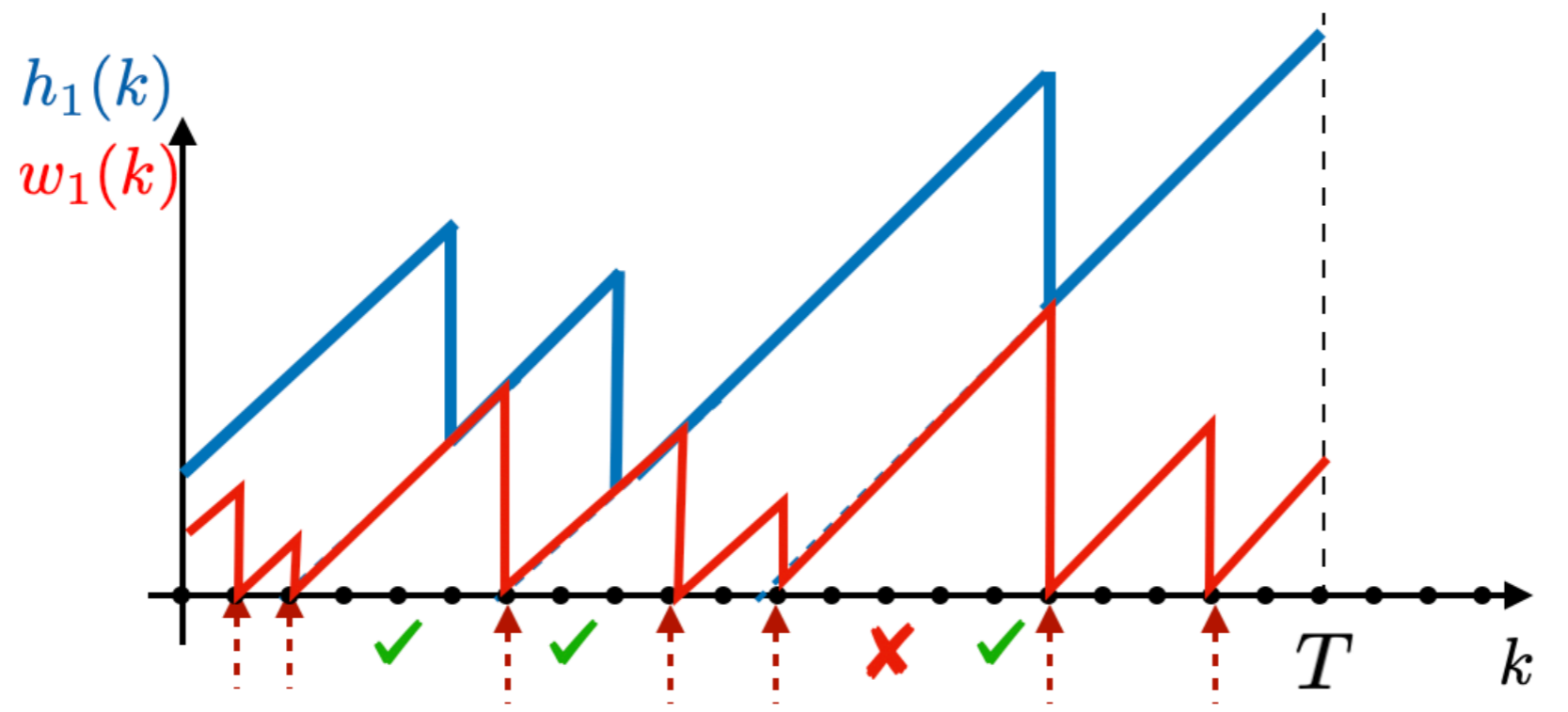
- Find transmission policy  $\pi$  that minimizes Normalized Expected Weighted Sum

$$\text{AoI (NEWSAoI)} \lim_{K \rightarrow \infty} \frac{1}{KM^2} \sum_{i=1}^M \sum_{k=1}^K h_i^{\pi}(k)$$

# Evolution of Age

**Source AoI**  $w_i(k+1) = \begin{cases} w_i(k) + 1 & \text{no new packet arrives} \\ 0 & \text{a new packet arrives} \end{cases}$

**Destination AoI**  $h_i(k+1) = \begin{cases} w_i(k) + 1 & \text{a packet is delivered} \\ h_i(k) + 1 & \text{no packet is delivered} \end{cases}$



# Lower Bound

Theorem: For any transmission policy, NEWSAoI is lower bounded by

$$1) \text{ NEWSAoI} \geq \frac{1}{M\theta} \quad \text{small arrival rates}$$

$$2) \text{ NEWSAoI} \geq \frac{1}{2C_{RA}} + \frac{1}{2M} \quad \text{large arrival rates}$$

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- RA with feedback  $C_{RA} \leq 0.568$  ( $M \rightarrow \infty$ )

[Tasybakov-Likhanov] *Probl. Peredachi Inf*, vol. 23

- RA with CSMA  $C_{RA} \leq 1$

- RA without feedback  $C_{RA} \leq \frac{1}{e}$  ( $M \rightarrow \infty$ )

# Decentralized Age-based Policies

Small arrival rate: slotted ALOHA

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Theorem: Suppose  $\theta \leq \frac{1}{eM}$  and define  $\eta = \lim_{M \rightarrow \infty} M\theta$ . Any stabilized slotted ALOHA scheme achieves

$$\lim_{M \rightarrow \infty} \text{NEWSAoI}(M) = \frac{1}{\eta}.$$

Moreover, (stabilized) slotted ALOHA are **asymptotically optimal** in terms of NEWSAoI.

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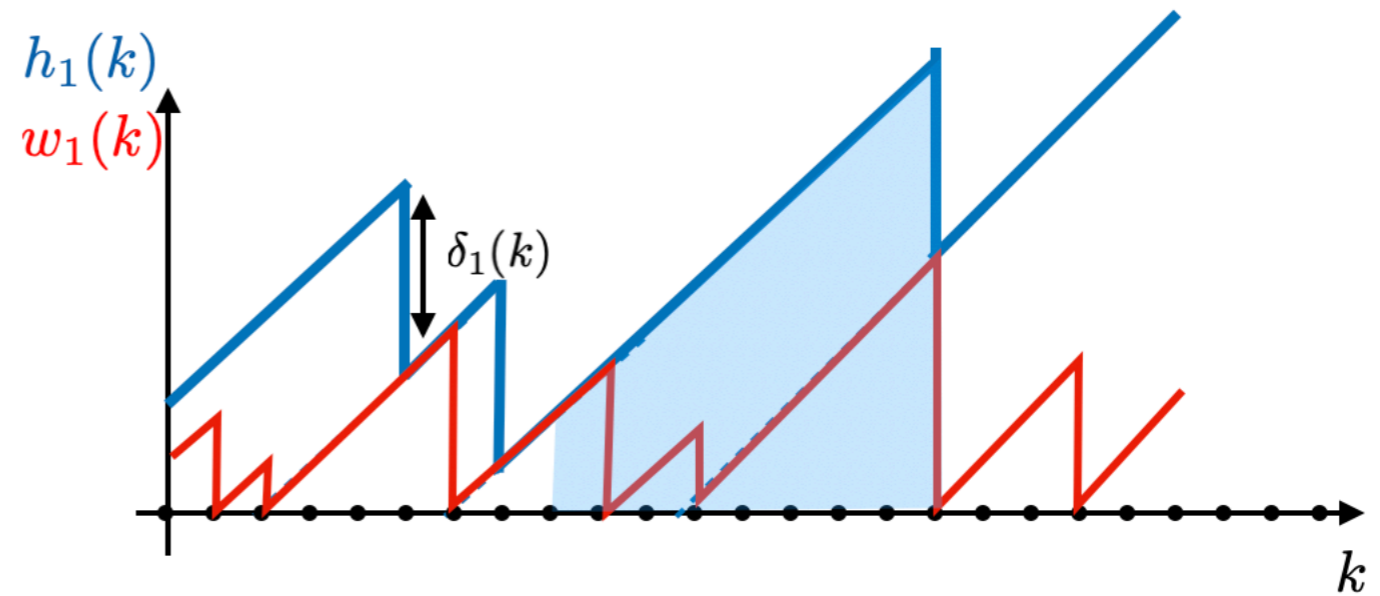
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- Is the slotted ALOHA unstabilized (large AoI) when  $\theta > \frac{1}{eM}$ ?
- Can we get benefits (small AoI) by increasing arrival rate?
- What should the transmitters do in order to ensure a small age of information when  $\theta > \frac{1}{eM}$ ?

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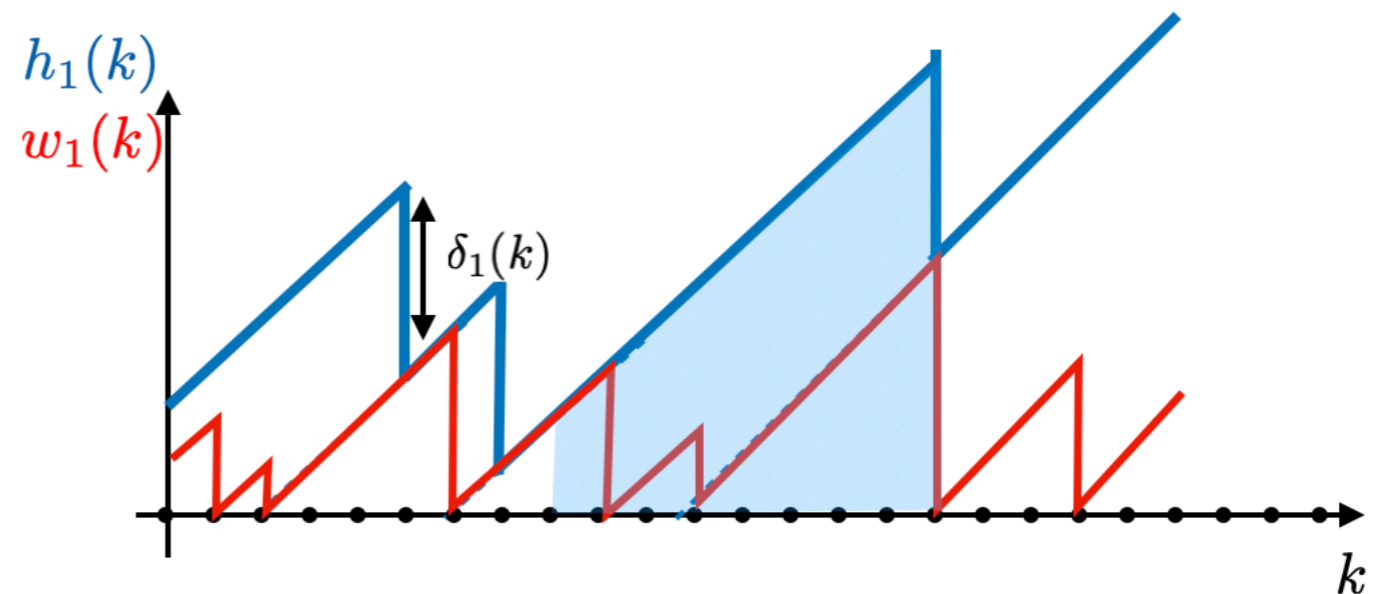
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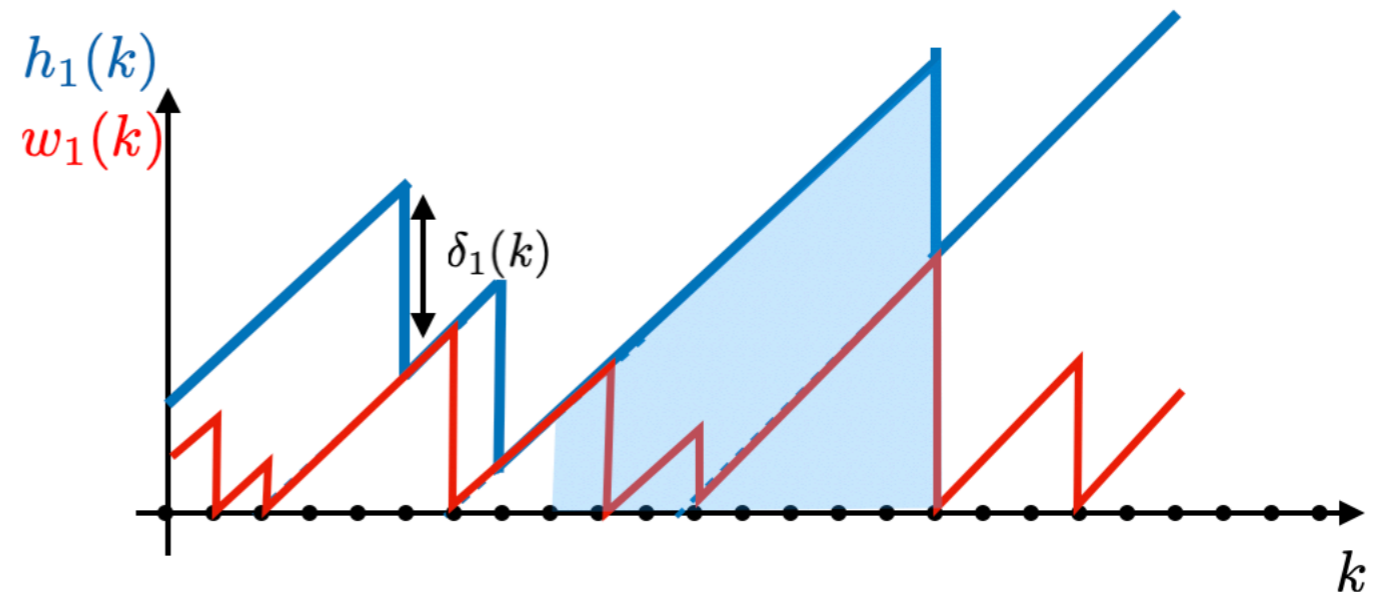


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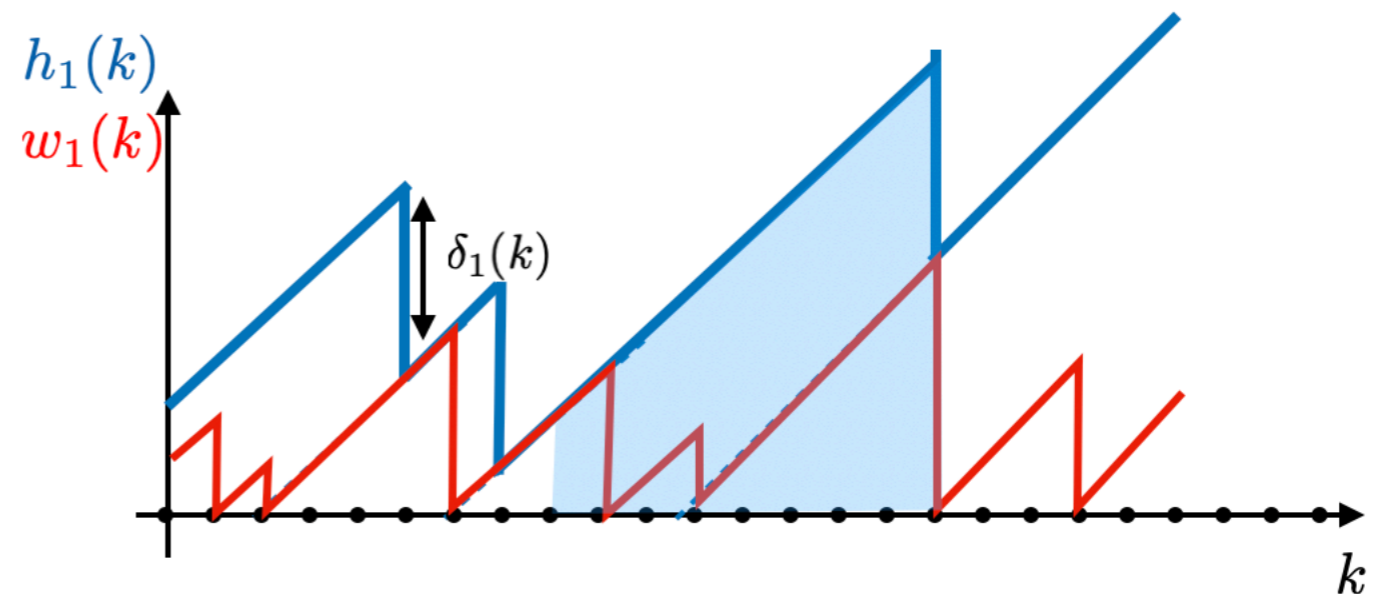
- **Adaptive** threshold policy: node  $i$ : 
$$\begin{cases} \text{active} & \delta_i(k) \geq T(k) \\ \text{inactive} & 0 \leq \delta_i(k) < T(k) \end{cases}$$

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- Active nodes follow slotted ALOHA protocol and inactive nodes remain silent

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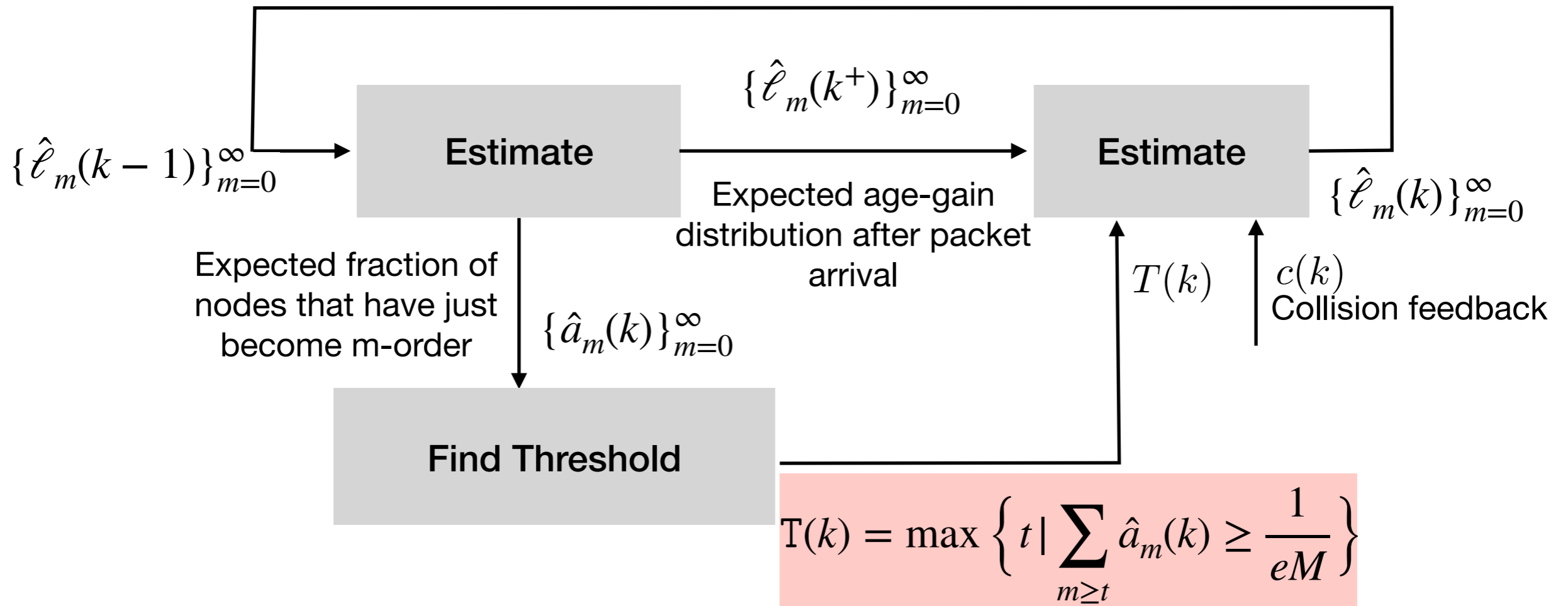
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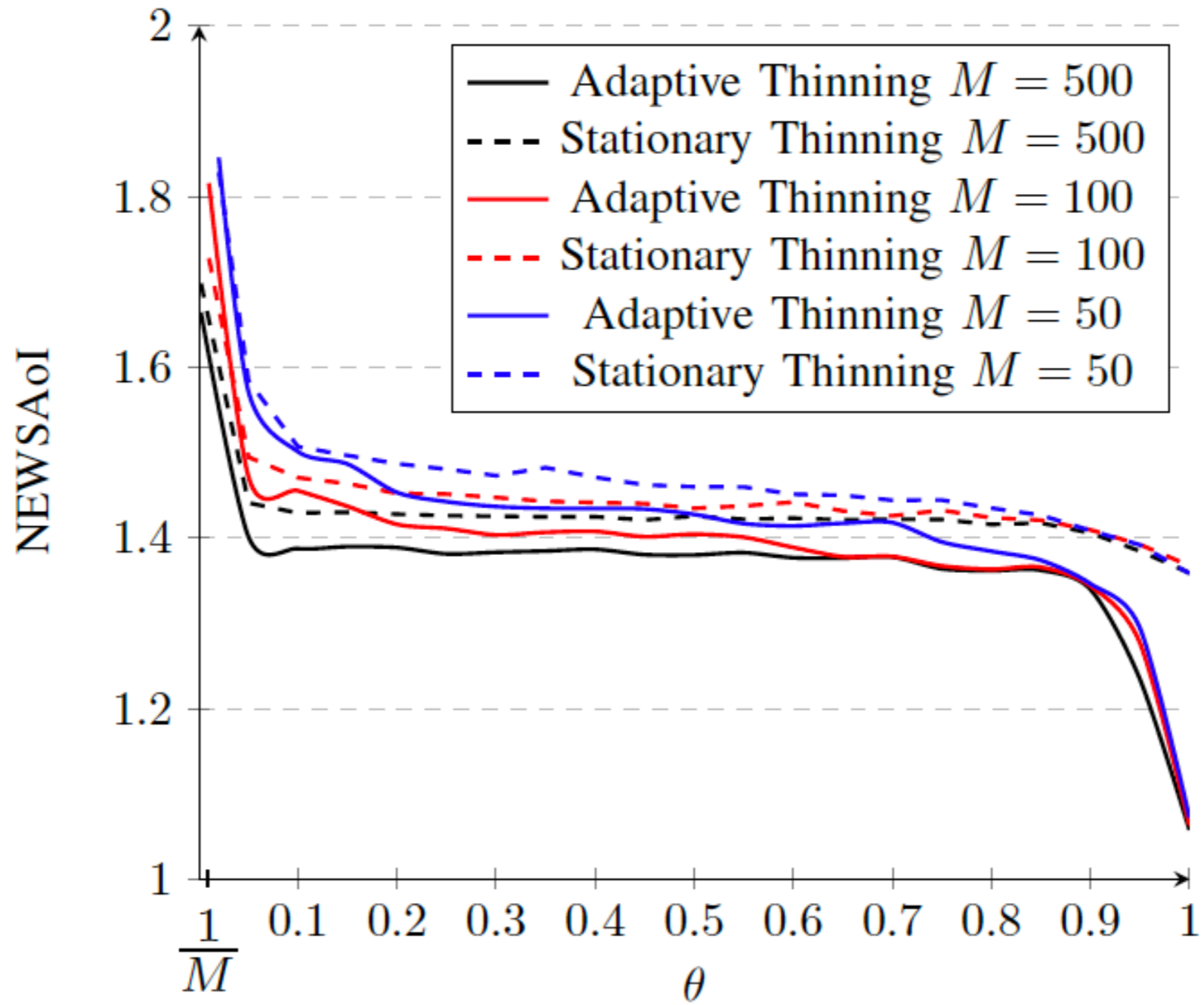
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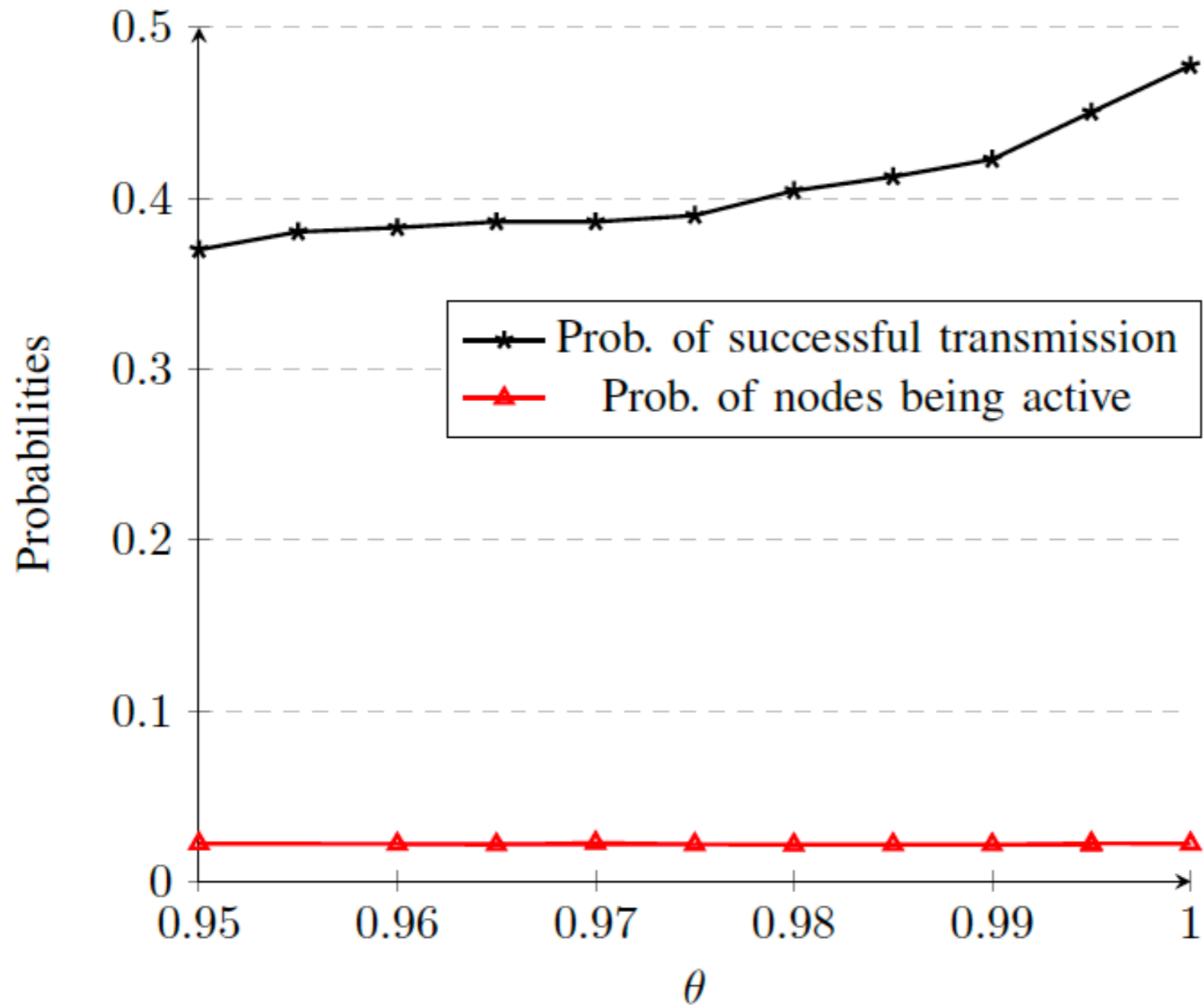
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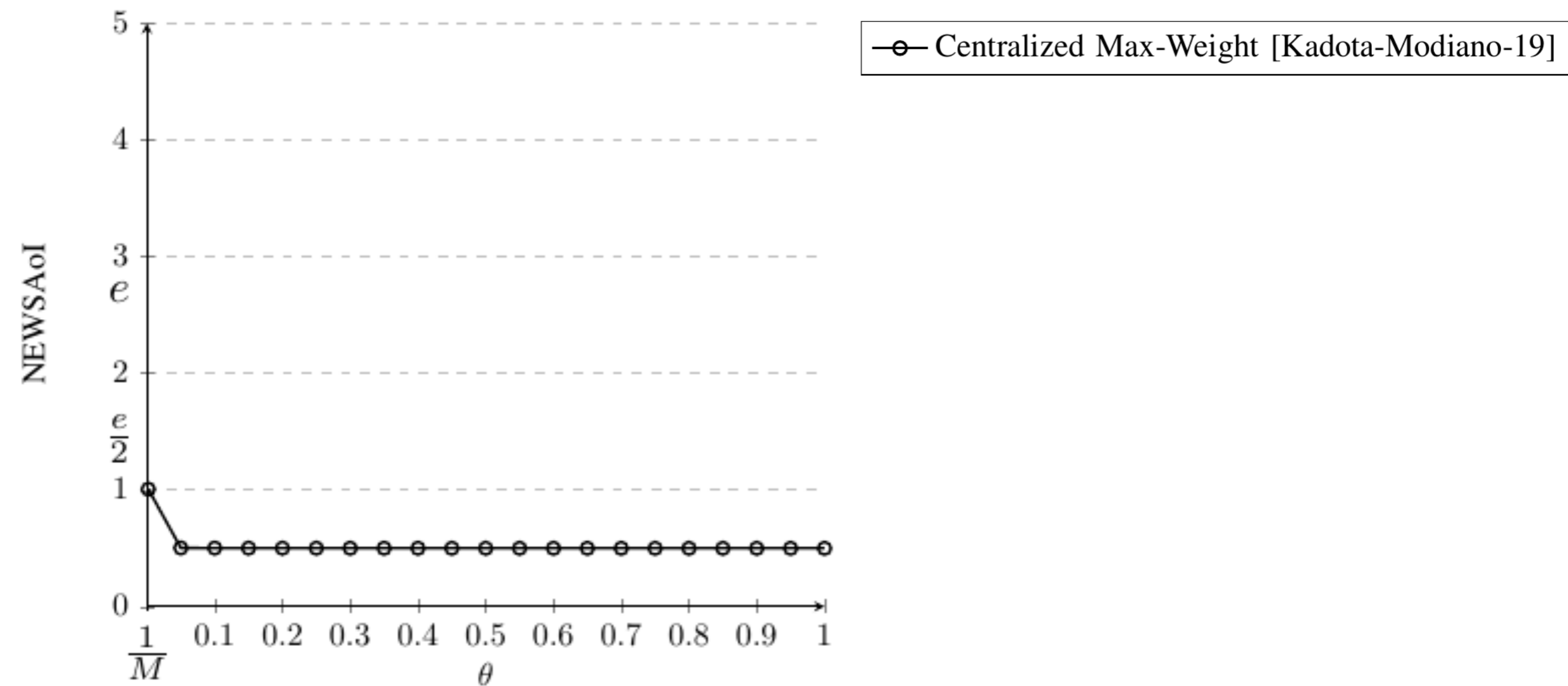


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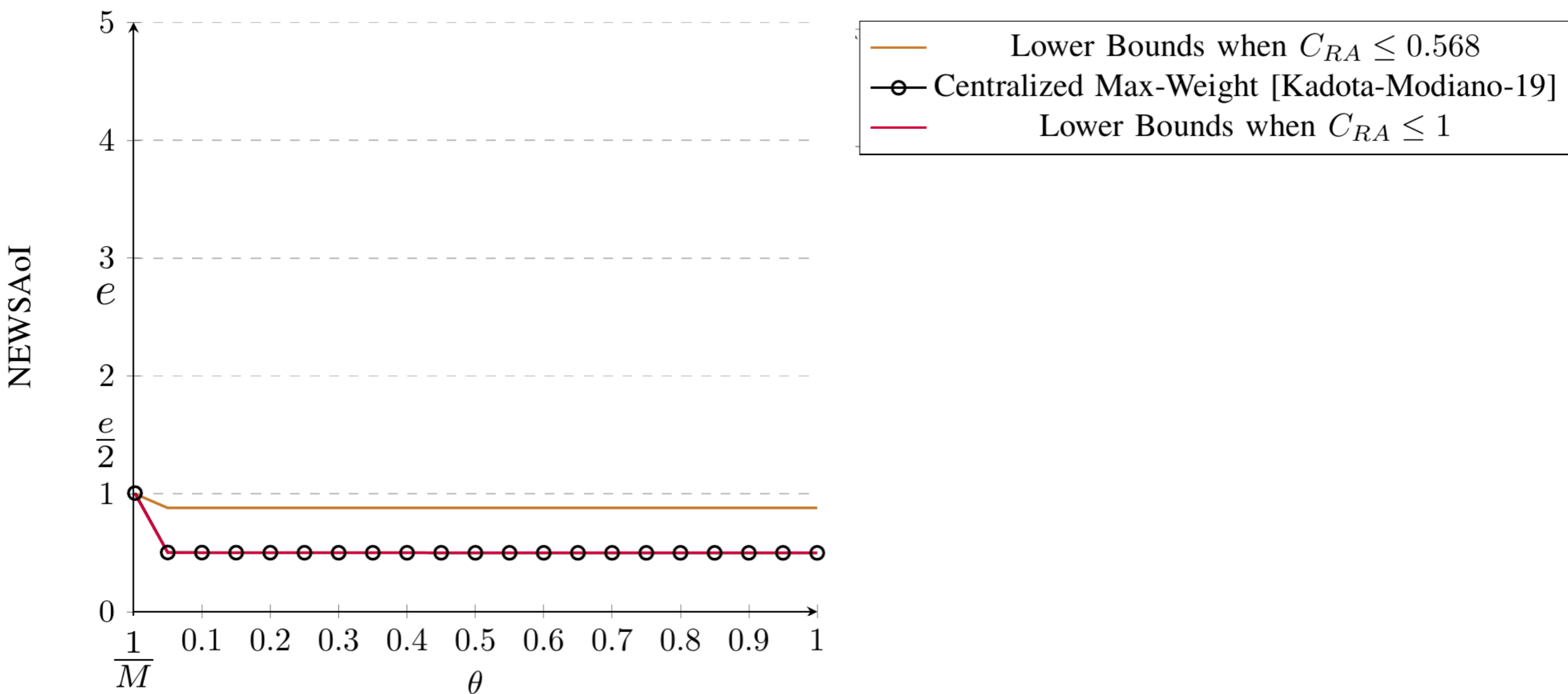
Throughput/rate  $\uparrow$   
Age of Information  $\downarrow$

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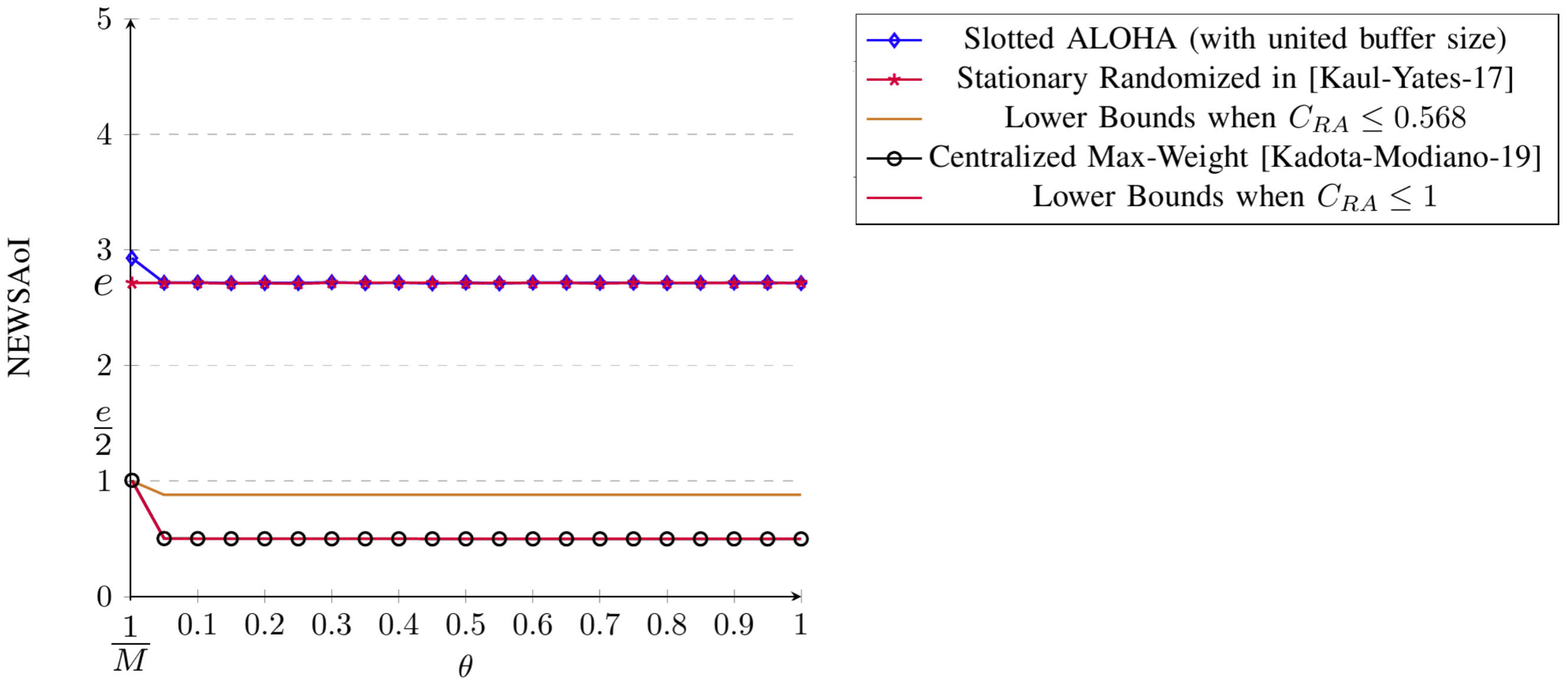
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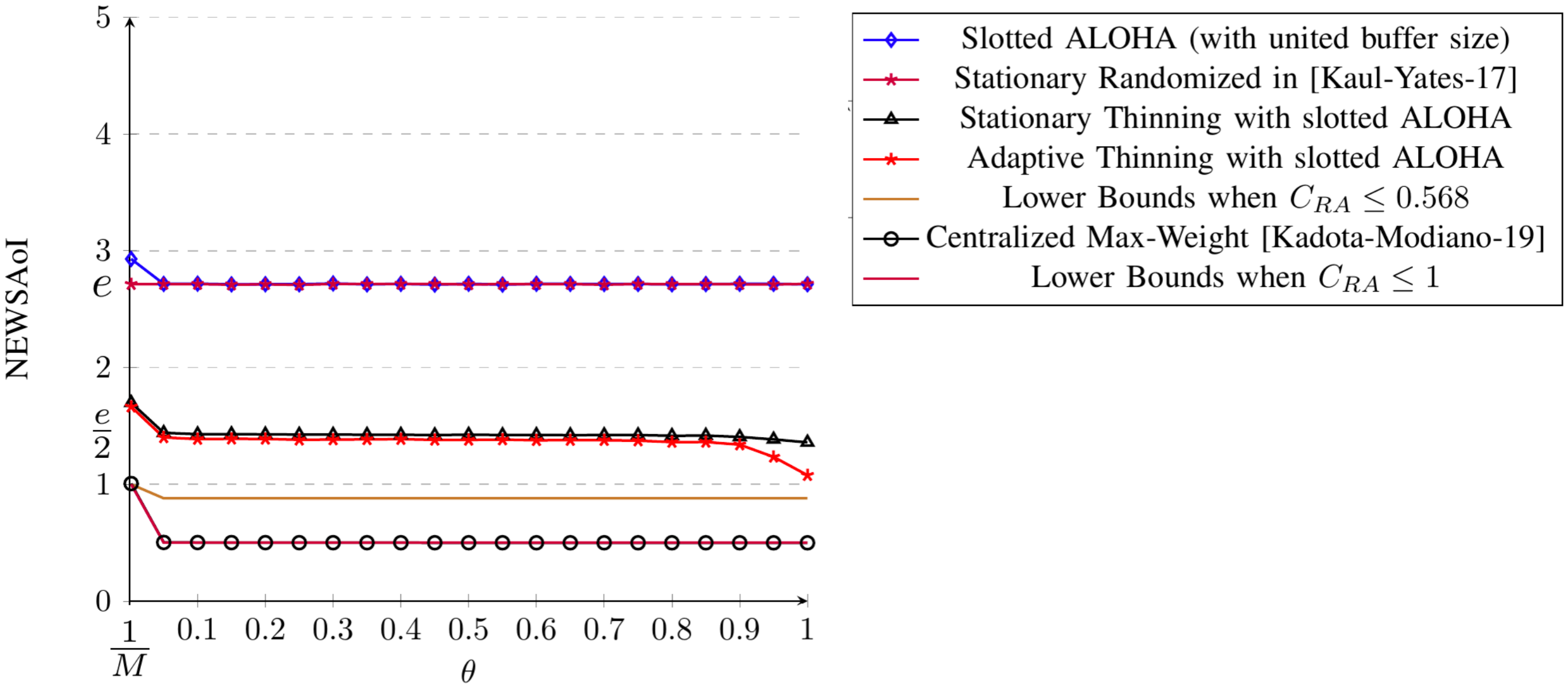
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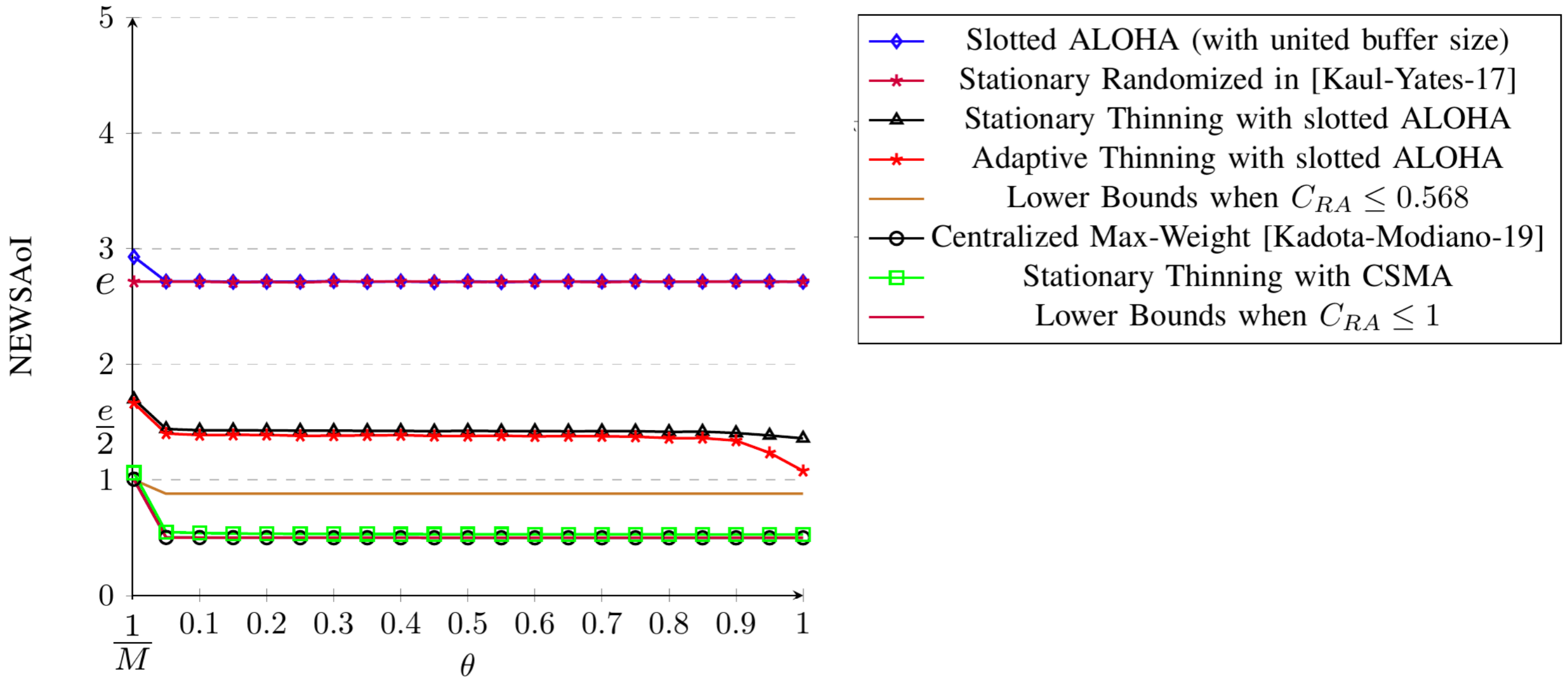
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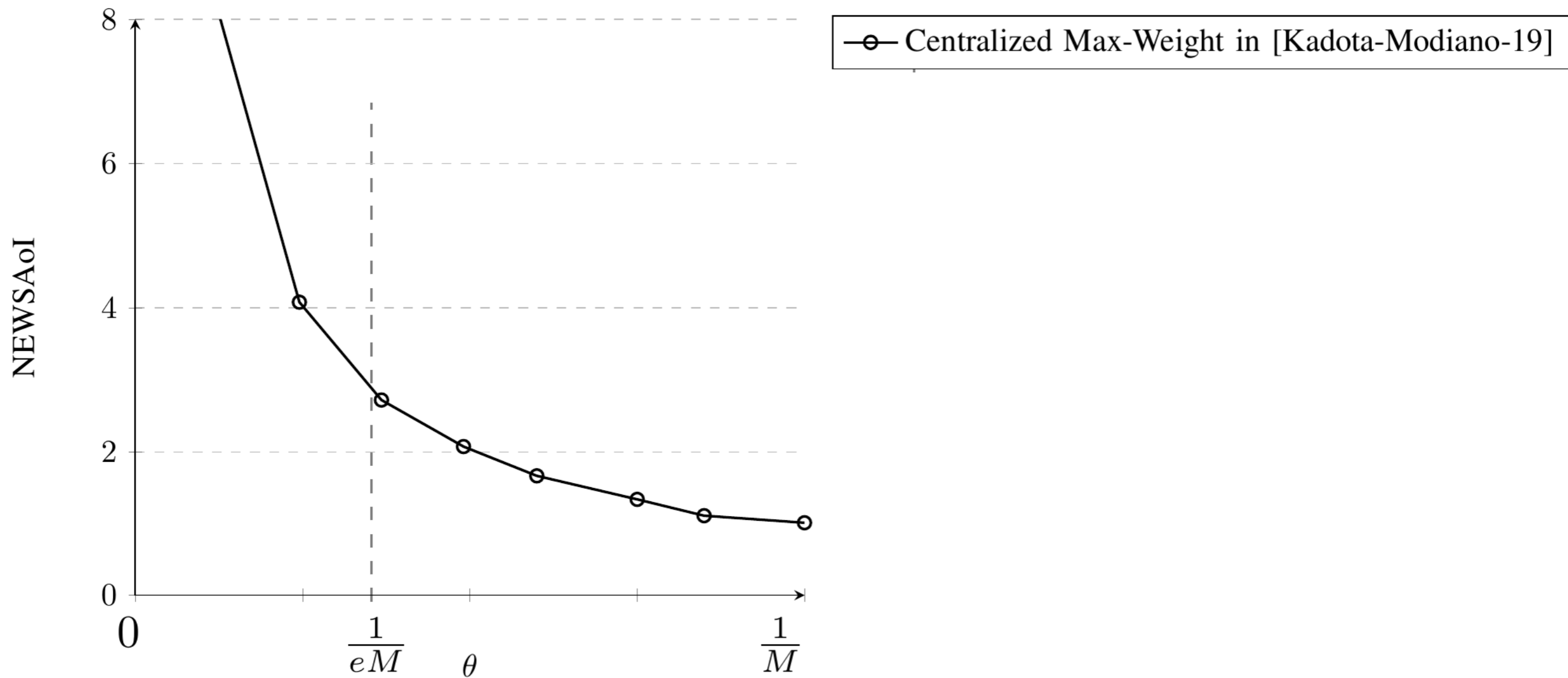
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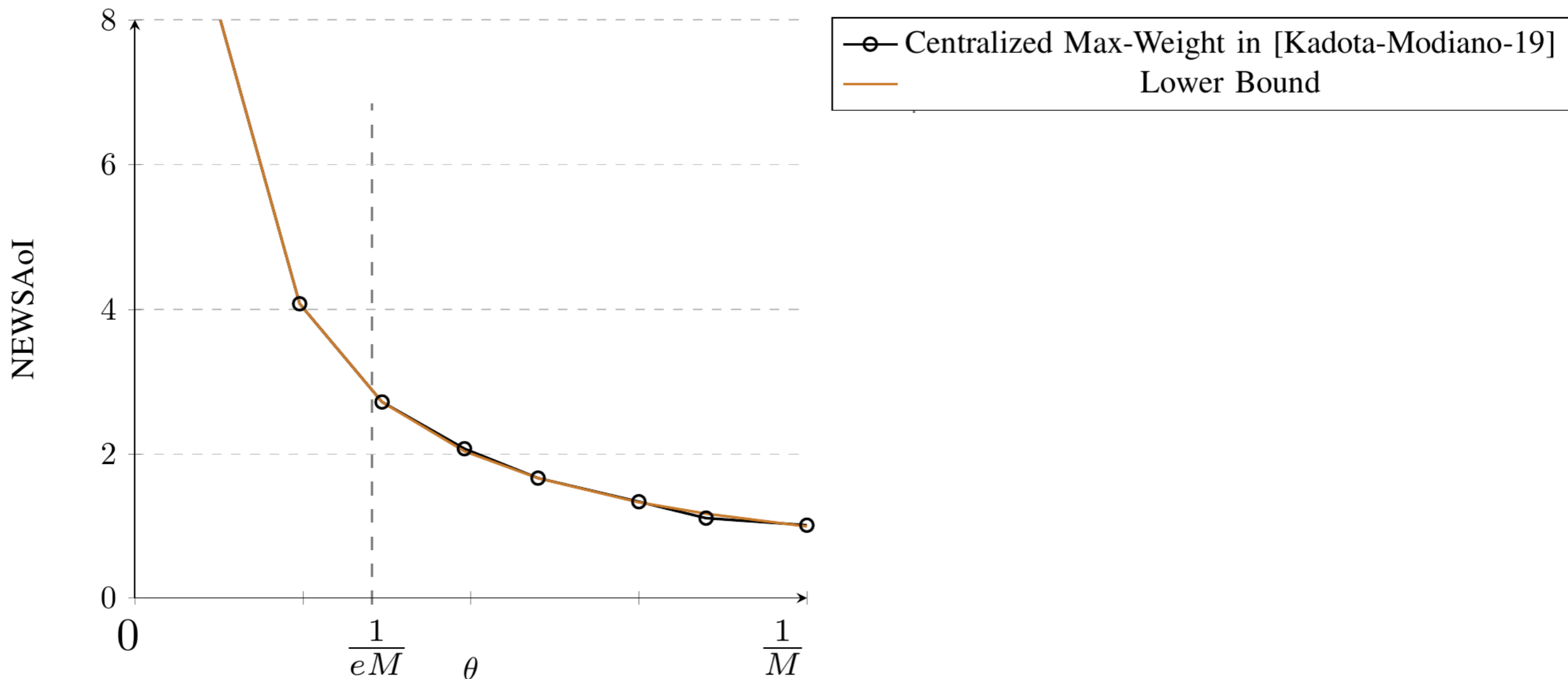


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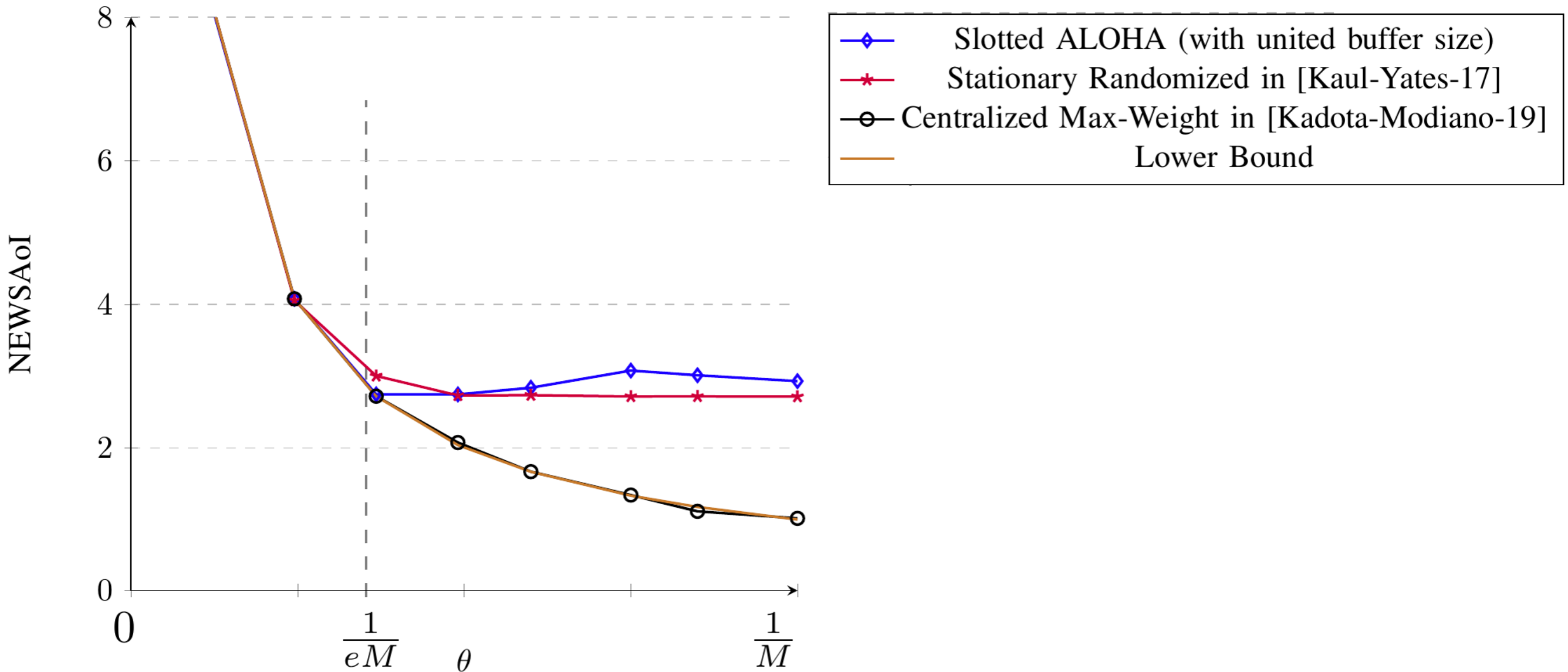
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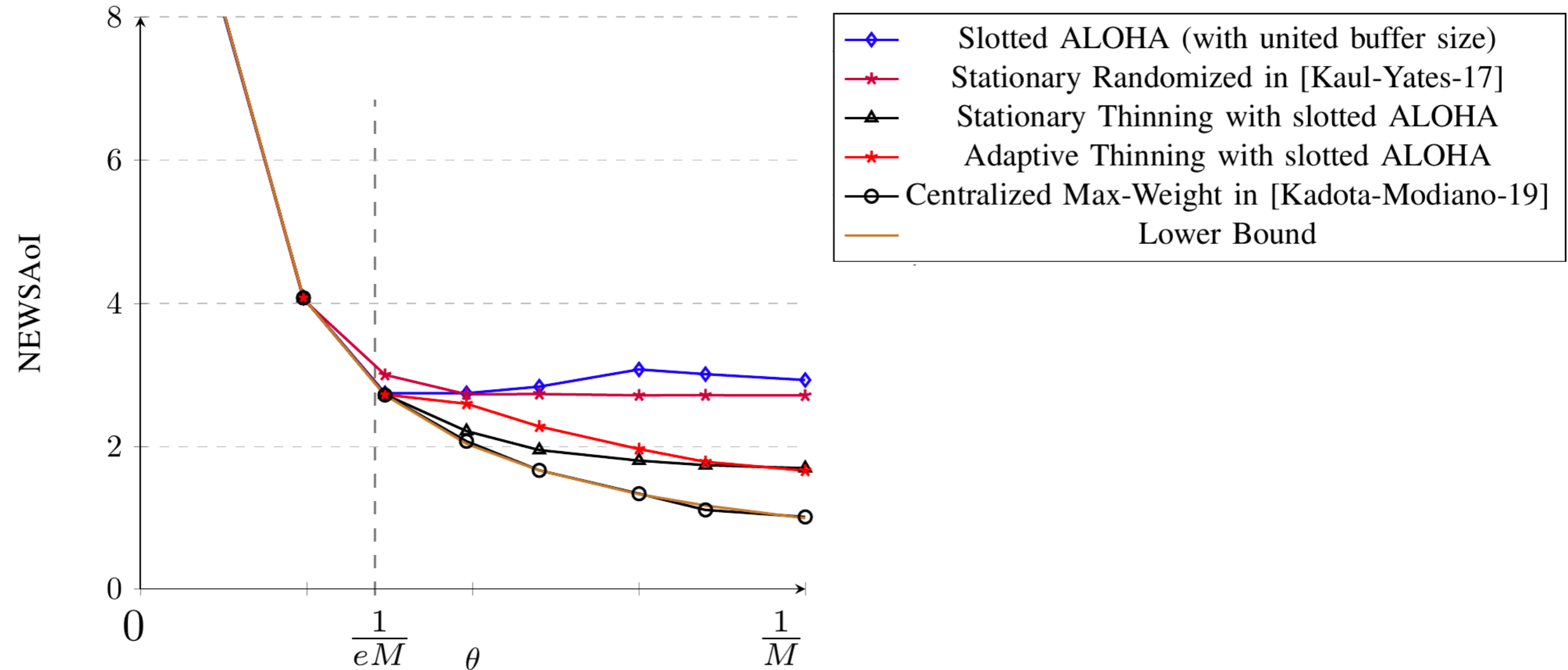
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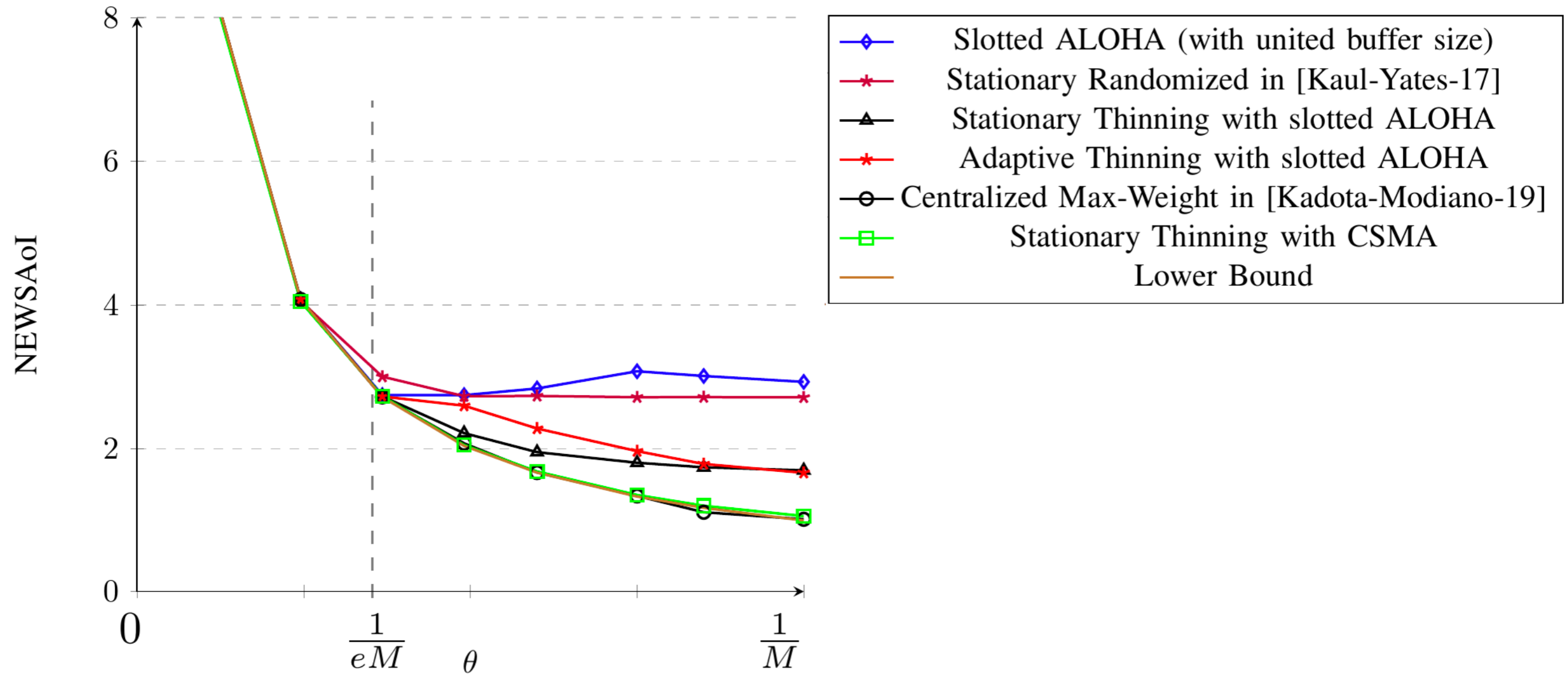
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