Real-time Sampling and Estimation on Random Access Channels: Age of Information and Beyond

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Background & Motivation

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- Remote estimation: various point-to-point scenarios

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- Reliable v.s. Timely Communication: the rate and/or reliability ↑ timeliness (Age of Information) ↓

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Decentralized sampling and remote estimation over a wireless collision channel

- Distributed decision making: each sensor decides when to sample and transmit information based only on its local observation

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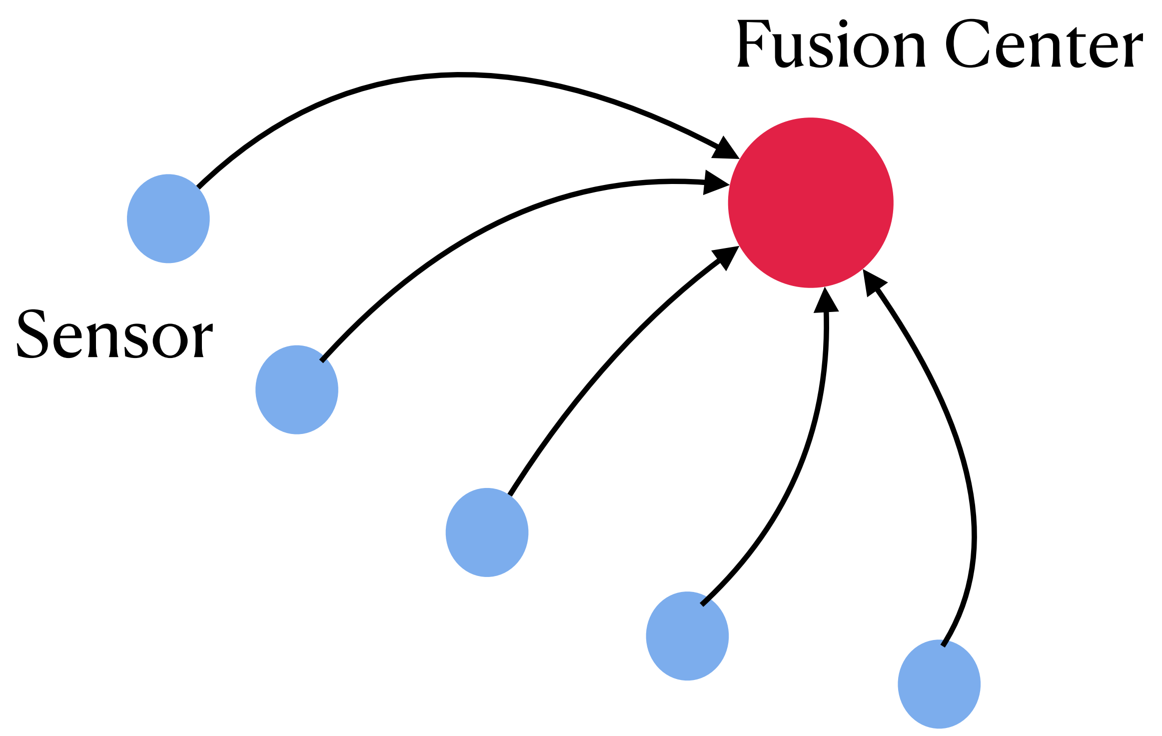
System Model



M statistically identical sensors and a fusion center

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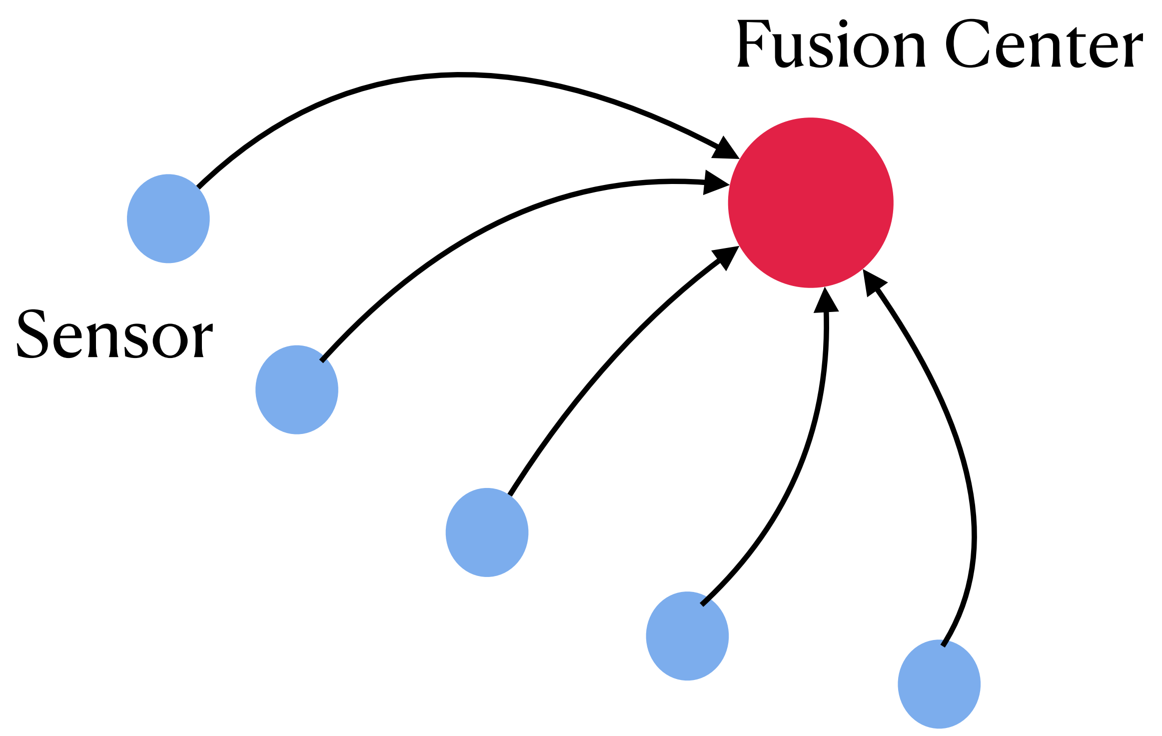

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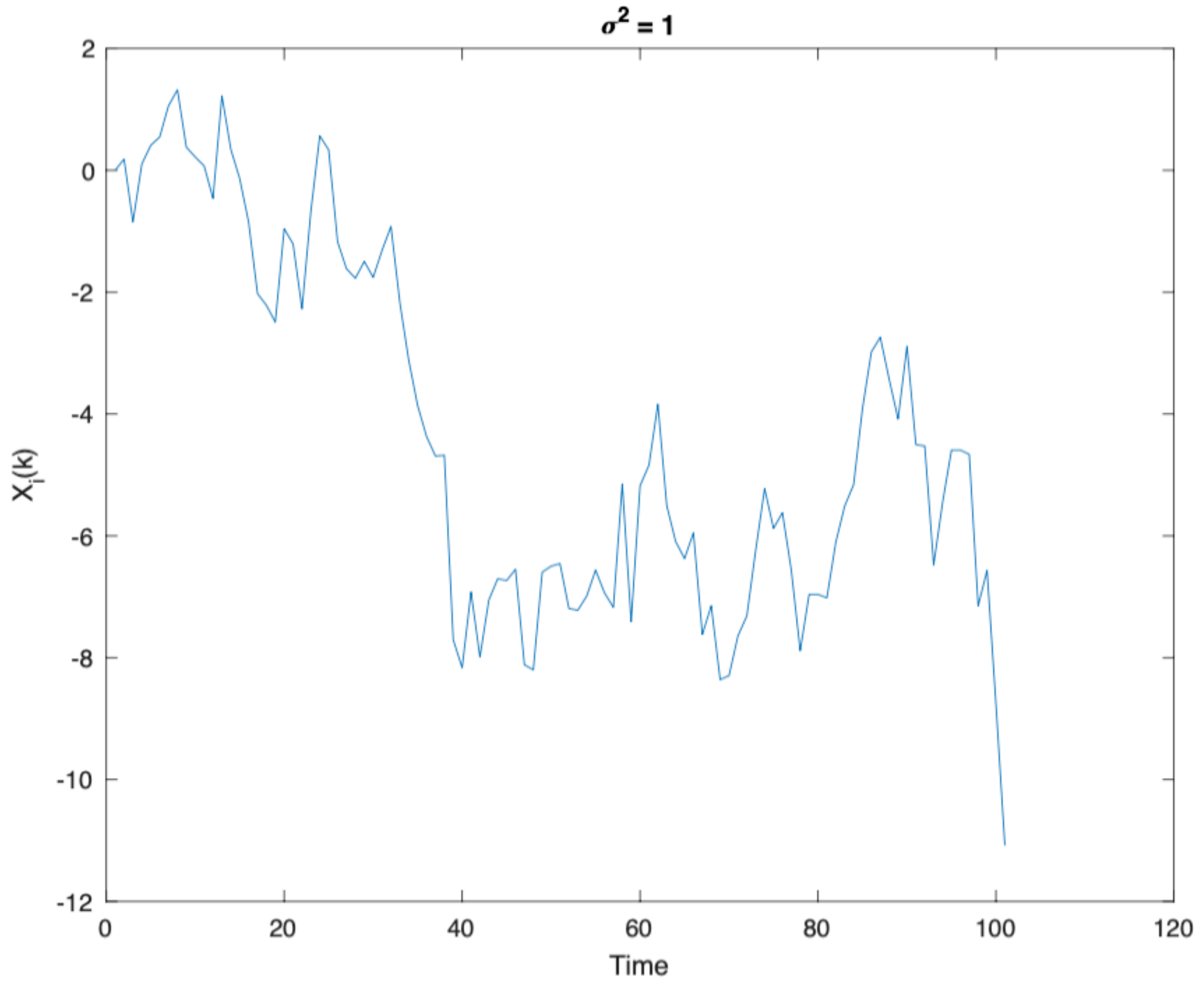

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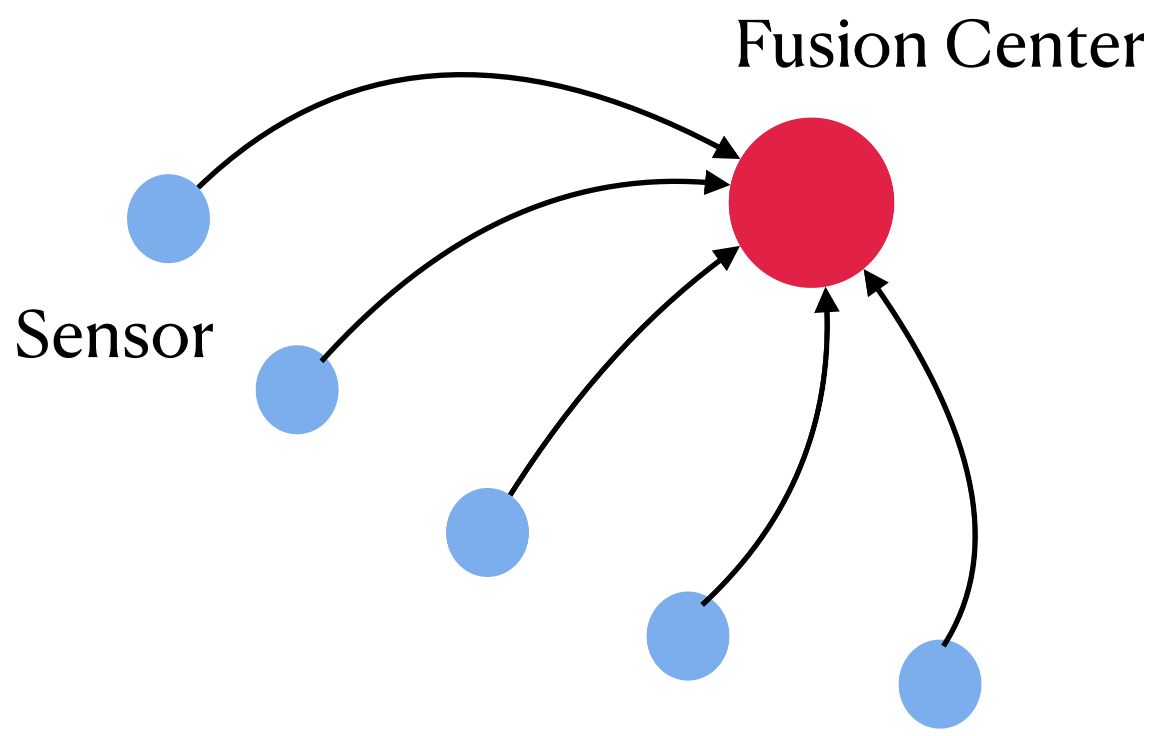

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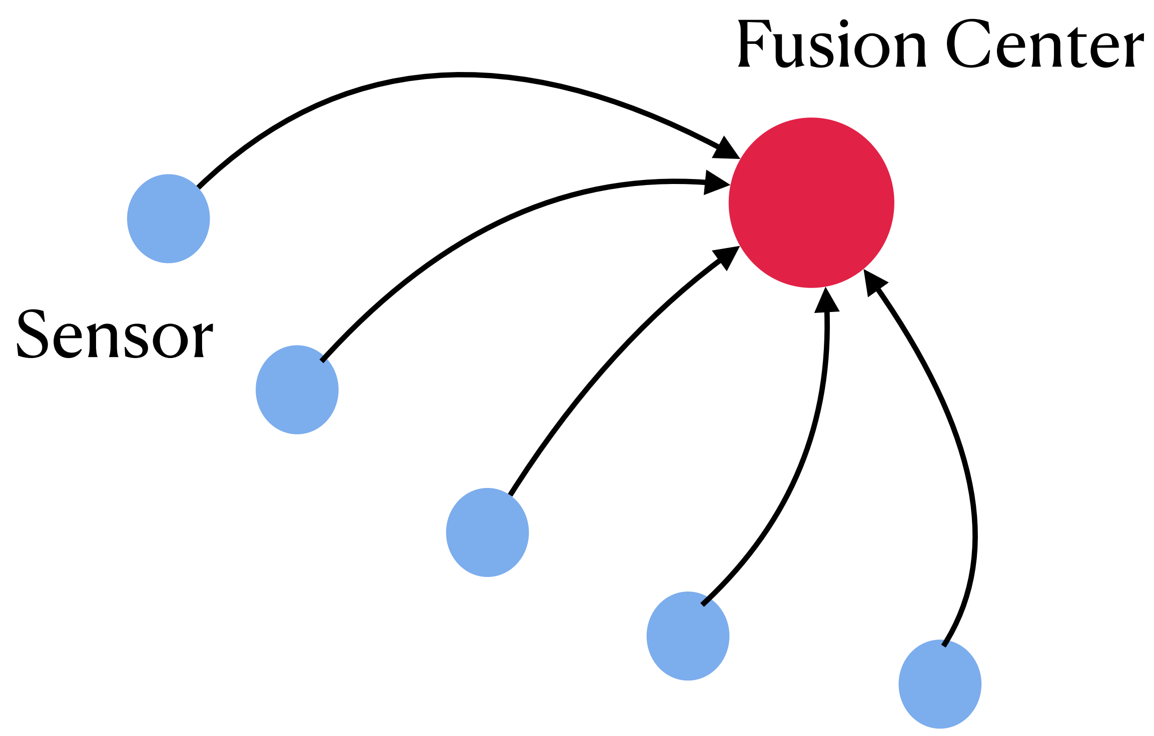

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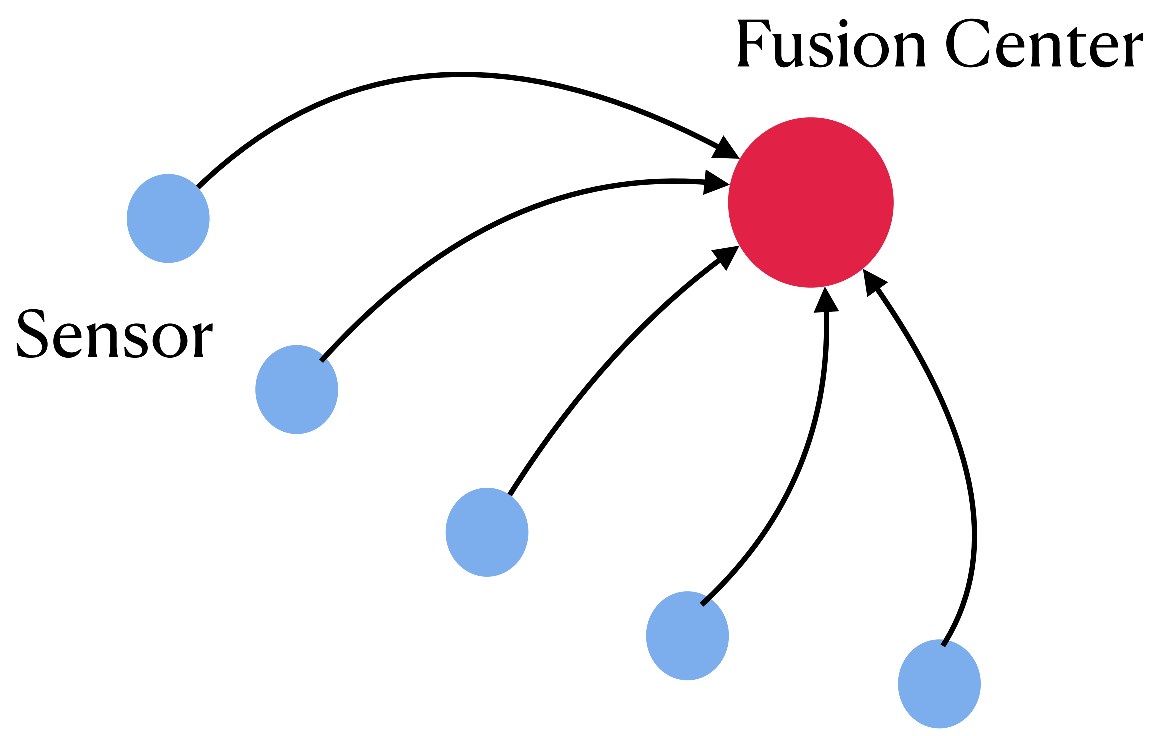

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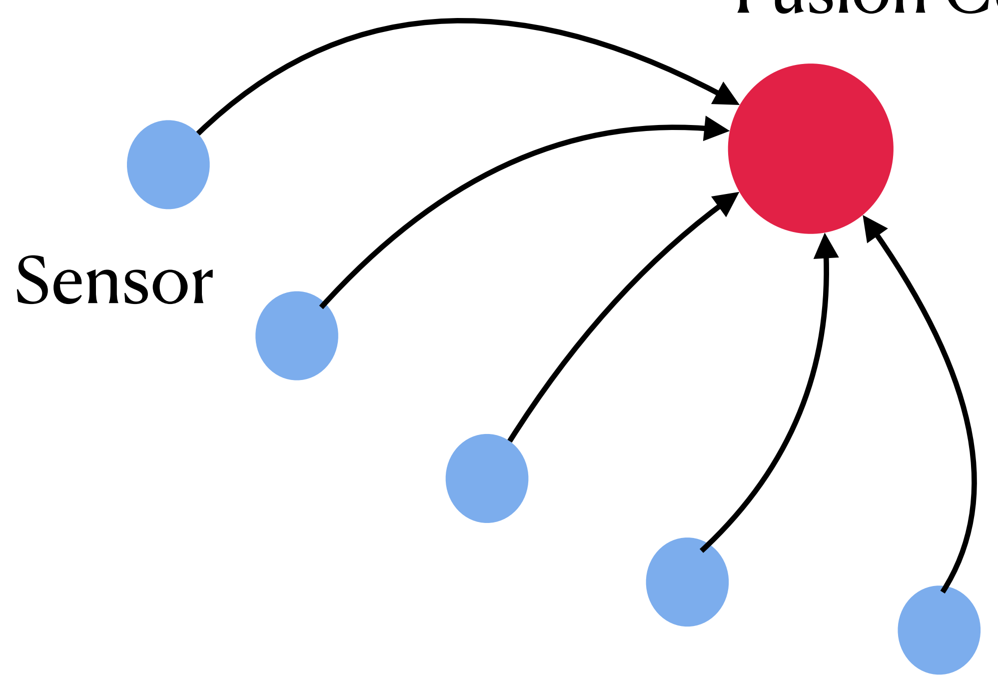

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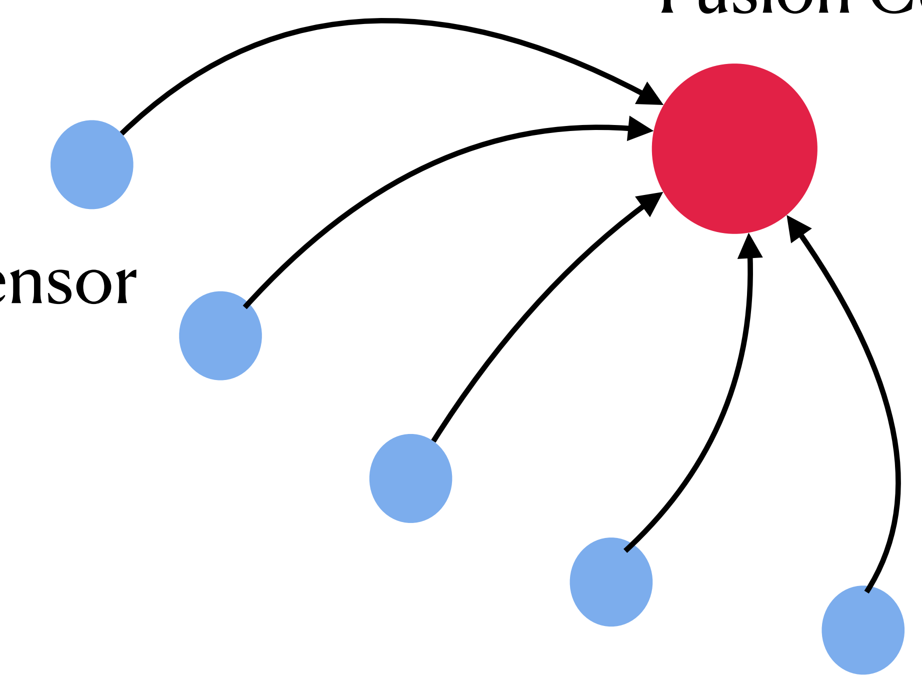

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Oblivious policies and non-oblivious policies

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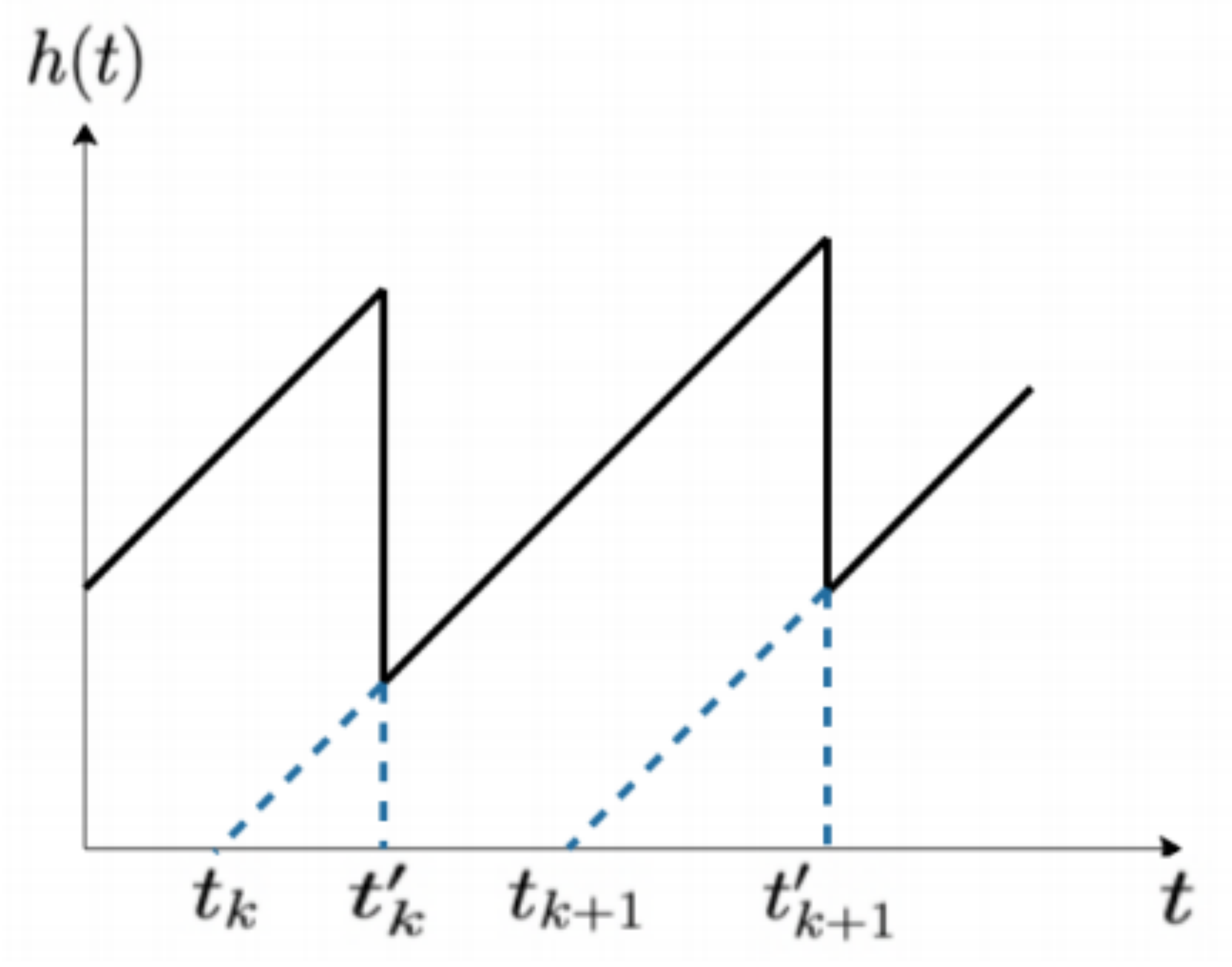

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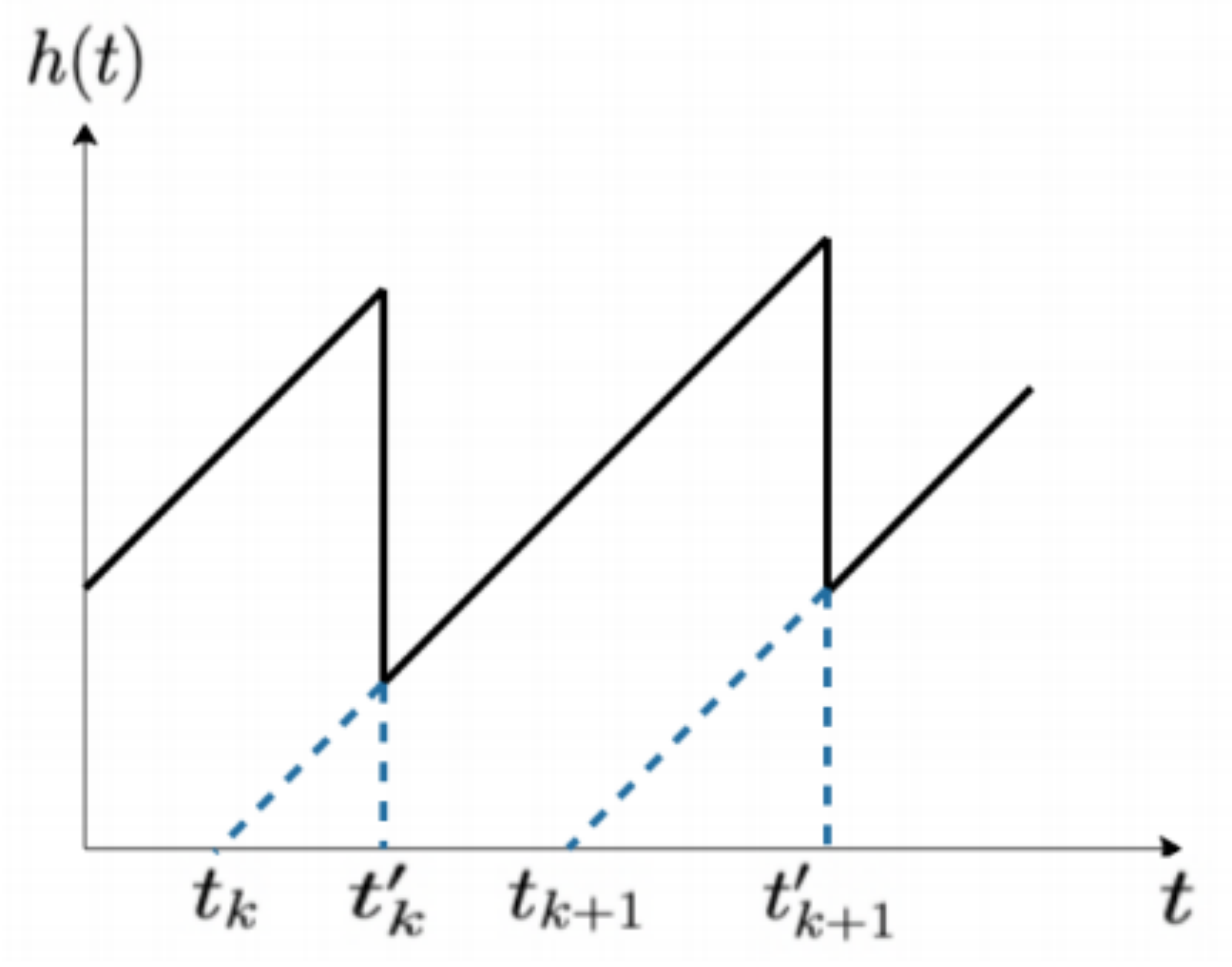

Lemma 1: In oblivious policies, the expected estimation error associated with process i has the following relationship with the expected age function: $\mathbb{E}\left[\left(X_i(k) - \hat{X}_i(k)\right)^2\right] = \mathbb{E}[h_i(k)]\sigma^2$.

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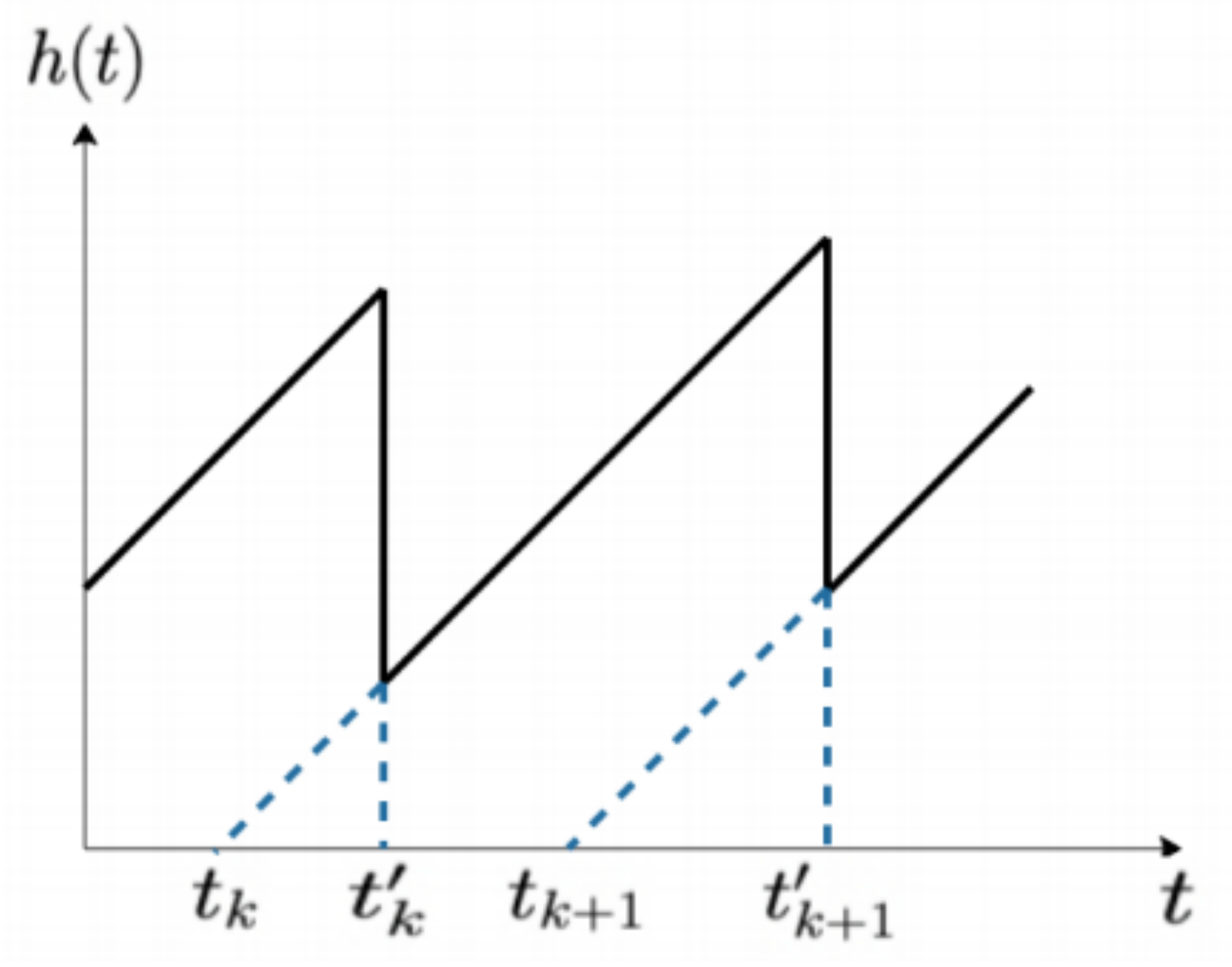

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Based on Lemma 1, $L^\pi(M) = \sigma^2 J^\pi(M), \quad J^\pi(M) = \lim_{K\to\infty} \frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K}\mathbb{E}[h_i^\pi(k)].$

$J^\pi(M)$ is the normalized expected sum of age of information,
which was investigated in our prior work [X. Chen - K. Gatsis - H. Hassani - S. Saeedi Bidokhti-2019]

Under SAT policy (Algorithm 2) in [X. Chen - K. Gatsis - H. Hassani - S. Saeedi-Bidokhti - 20],

$$\lim_{M\to\infty} J^{SAT}(M) = \frac{e}{2}, \quad \lim_{M\to\infty} L^{SAT}(M) = \frac{e}{2}\sigma^2$$

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Under the Max-Weight policy, $\lim_{M\to\infty} L^{MW}(M) = \frac{\sigma^2}{2}$, which implies $\lim_{M\to\infty} \frac{L^{SAT}(M)}{L^{MW}(M)} = e$

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All nodes transmit stochastically following Rivest's stabilized slotted ALOHA

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Error-based Thinning (EbT); Find an optimal threshold β

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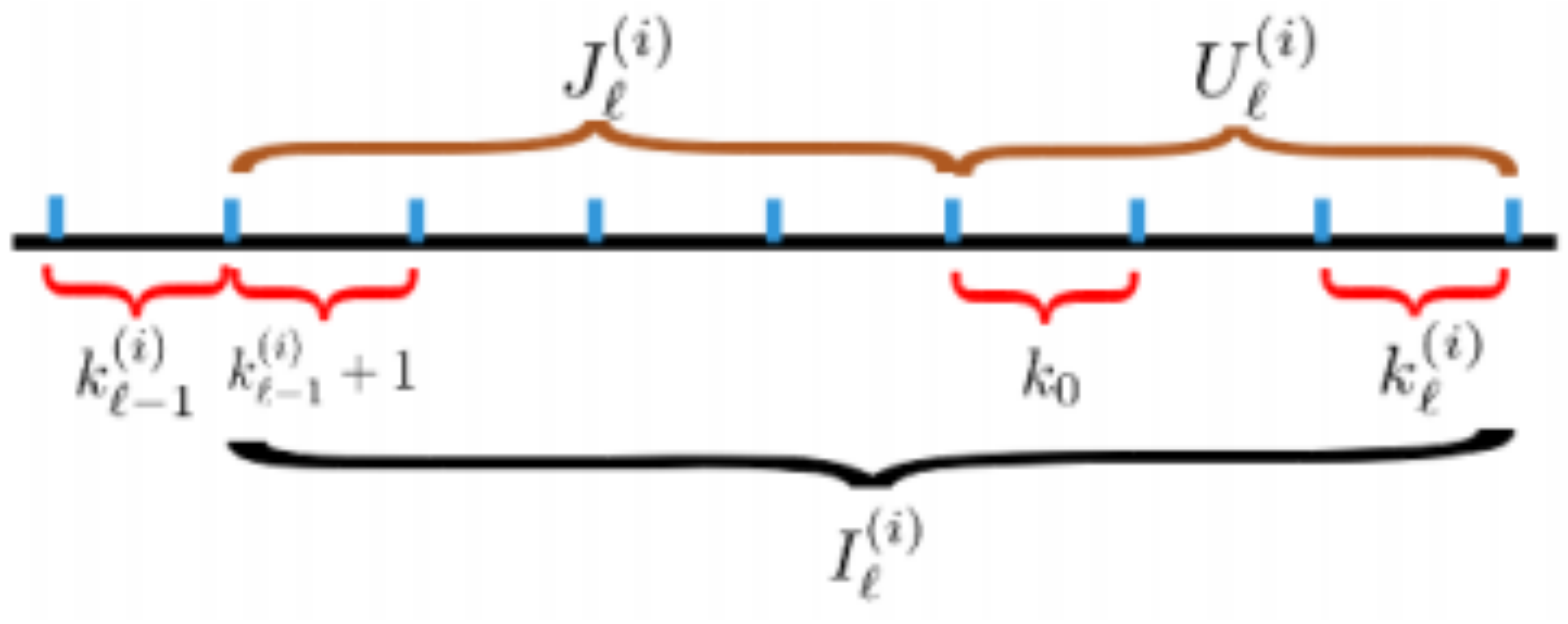
Definition 3: Define $J_l^{(i)} = k_0 - k_{l-1}^{(i)}$ as the silence delay.
Define $U_l^{(i)} = k_l^{(i)} - k_0 + 1$ as transmission delay. $I_l^{(i)} = J_l^{(i)} - 1 + U_l^{(i)}$.



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When M is sufficient large. By some algebra, $L^{EbT}(M) = \frac{1}{M}\frac{\mathbb{E}\left[\sum_{j=1}^{J_\beta} S_j^2\right]}{\mathbb{E}[I_\beta]} + \frac{1}{M}\frac{\mathbb{E}[U_\beta^2]}{\mathbb{E}[I_\beta]}$

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J_β: Stopping time

Propose to use Brown motion B_j as an approximation of S_j/σ.

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The estimate of $L^{EbT}(M)$ is $\hat{L}^{EbT}(M) = \dfrac{\frac{1}{5}\mathbb{E}[J_\beta^2] + \mathbb{E}[U_\beta^2]}{2M\mathbb{E}[I_\beta]}\sigma^2$.

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Estimation Error Analysis: the approximation error in $L^{EbT}(M)$ increases linearly with σ^2.

Numerical Results

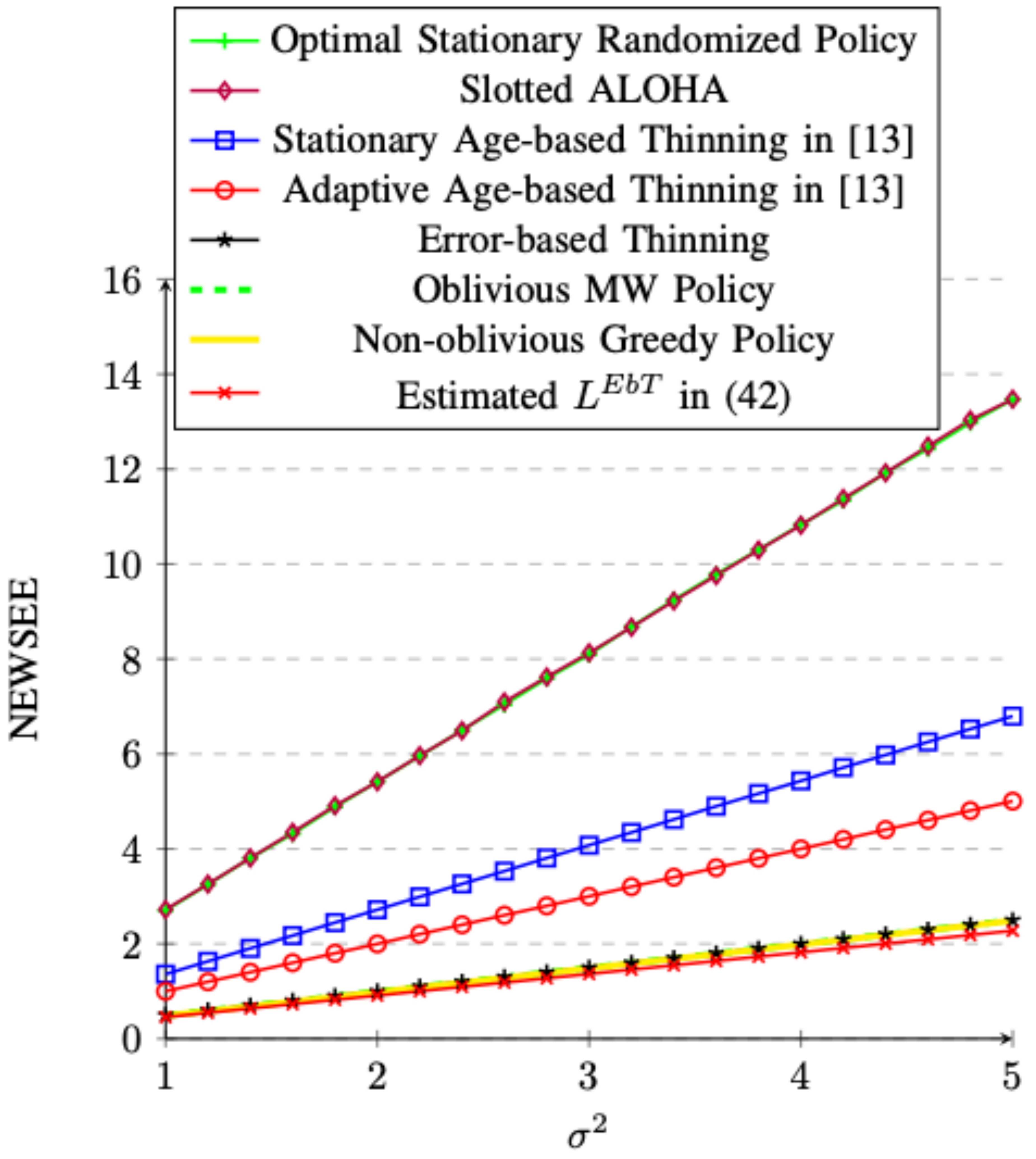

Fig. 2: NEWSEE as a function of σ^2 for various state-of-the-art scheme with $M = 500$.

Numerical Results

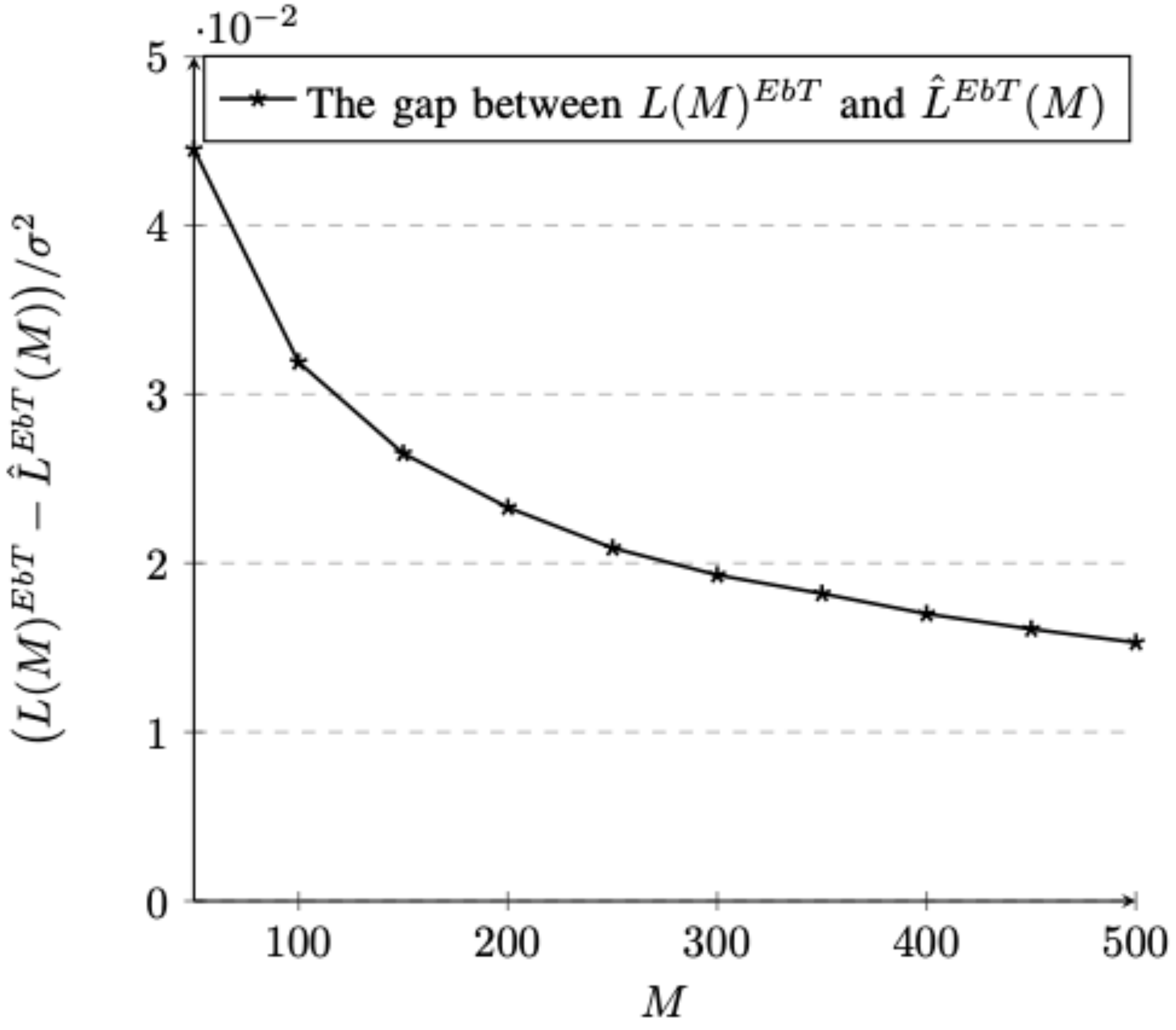

Fig. 6: The gap (normalized by σ^2) between $L^{EbT}(M)$ and $\hat{L}^{EbT}(M)$ as a function of M for $\sigma^2 = 3$.

Thank you!