Timely Broadcasting in Erasure Networks: Age-Rate Tradeoffs

2021 IEEE ISIT, Melbourne, Victoria, Australia

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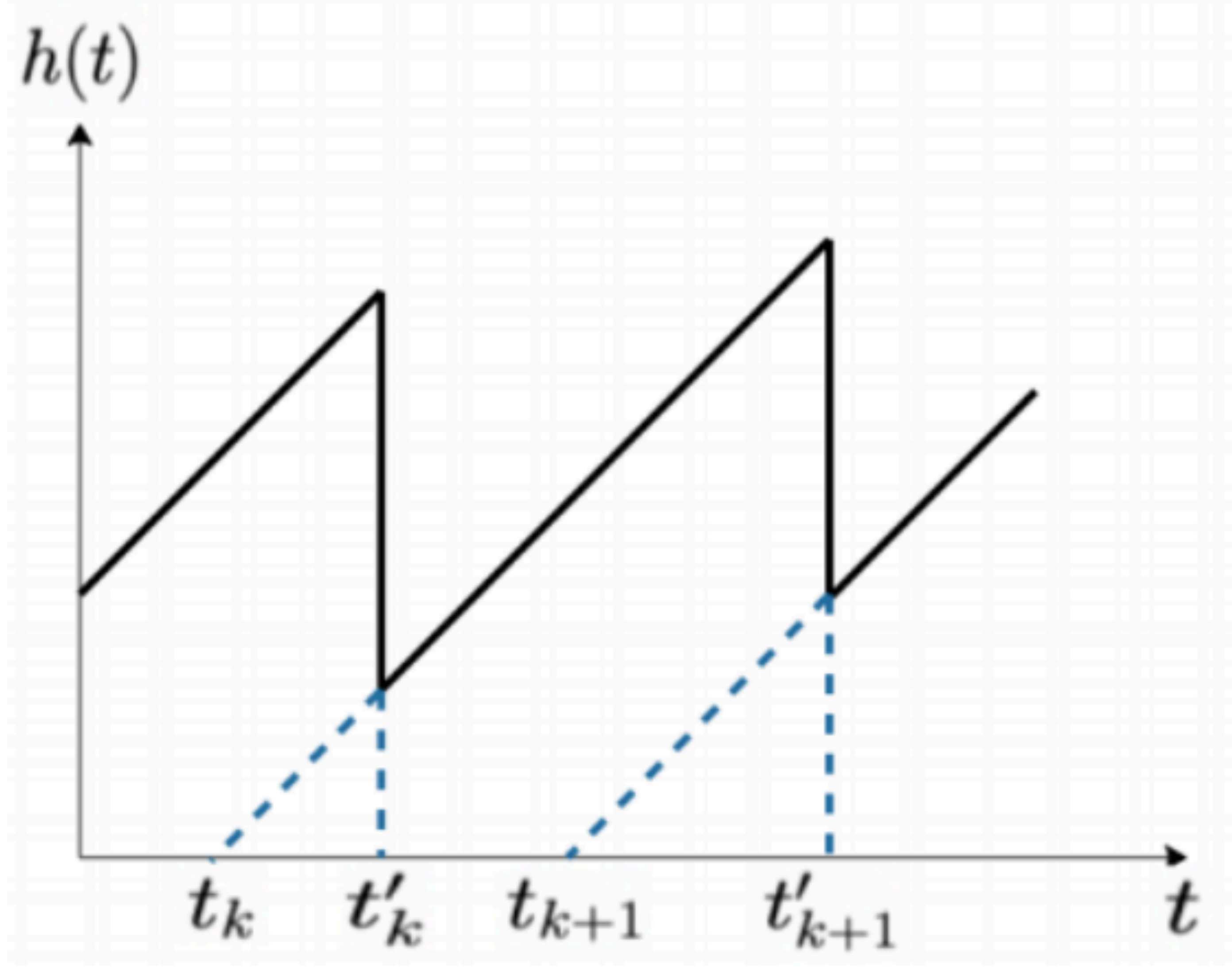
t_k: the generation time of k^{th} status update

Time average age: $\lim_{T\to\infty} \frac{1}{T}\int_0^T h(t)dt$



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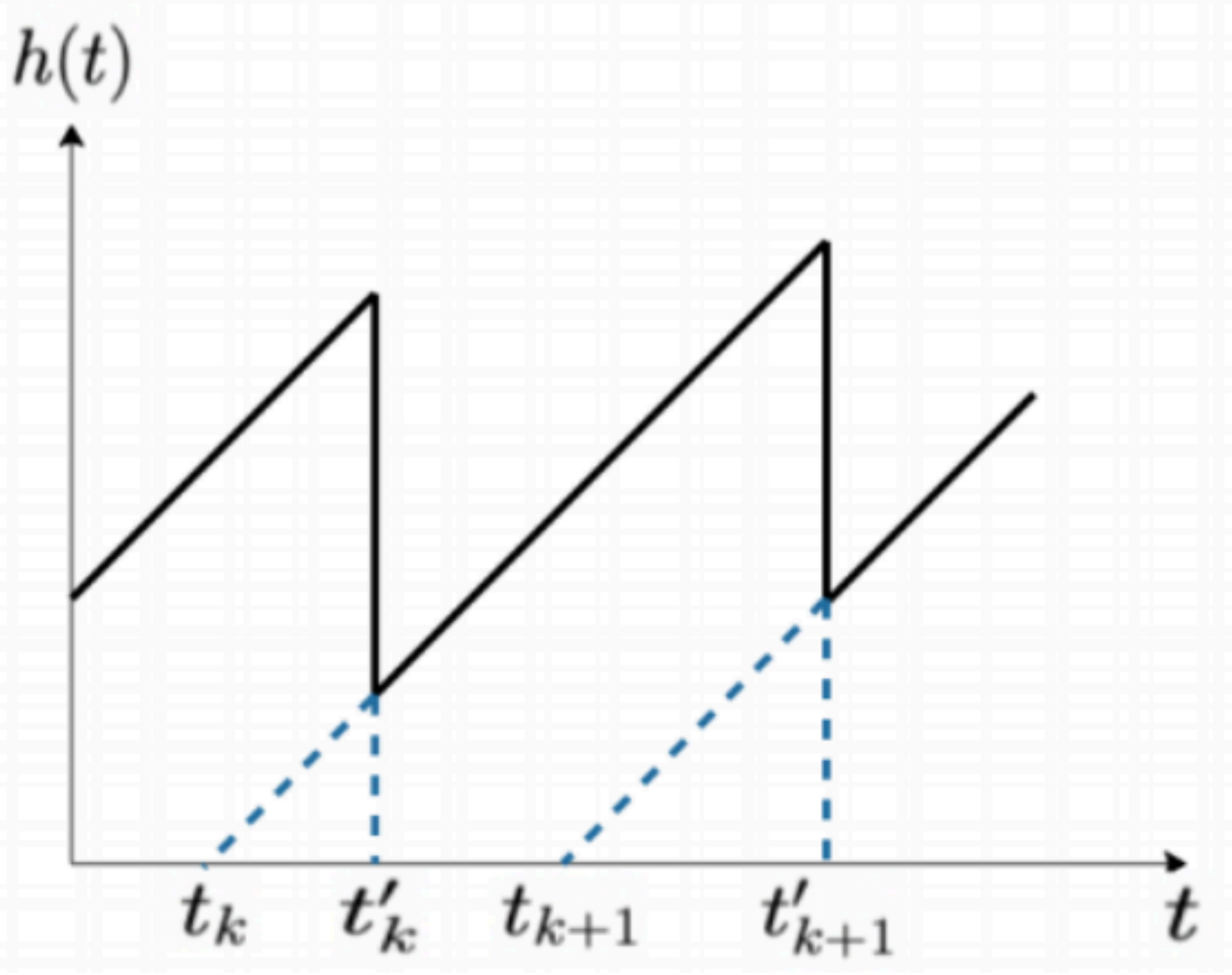

Rate efficiency: is often provided by channel coding schemes; cost of large delays; not clear a-priori what types of tradeoffs exist between rate and timeliness.

[Chen-Huang-16], [Yates-Najm-Soljanin-Zhong-17], [Parag-Taghavi-Chamberland-17], [Najm-Telater-Nasser-19], [Sac-Bacinoglu-Biyikoglu-Durisi-18], [Feng-Yang-19, 20], [Costa-Sagduyu-19]

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We build on our previous work [20] and consider an erasure wireless network with M users.

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We consider three type policies: (i) policies that benefit from coding by caching uncoded packets, (ii) policies that benefit from coding by caching general packets, (iii) time-sharing policies.

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More generally, consider a set of non-empty queues $\{Q_{\tau_i,\mathcal{S}_{\tau_i}}\}_{i=1}^{\ell}$. If

$$\mathcal{S}_{\tau_i} \supset \{\cup_{j=1,j\neq i}^{\ell}\tau_j\} \quad i \in \{1,2,\cdots,\ell\} \tag{1}$$

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Side information graph: add an edge between nodes (i,j) if $Q_{i,\mathcal{S}_i}$ is non-empty for $\mathcal{S}_i$ that has j as an element. Condition (1) corresponds to the subgraph induced by nodes $\{\tau_1, \cdots, \tau_\ell\}$ forming a clique of size ℓ.

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It is sufficient to consider all possible maximal cliques.

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$$J(\underline{q}) := \min_{\pi}\lim_{K\to\infty}\mathbb{E}[J_K^\pi]$$

$$s.t. \quad r_i^\pi \geq q_i, \quad i \in \{1,2,\cdots,M\}$$

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Define the age of information of $Q_{i,\mathcal{S}}$ as $w_{i,\mathcal{S}}(k)$.

Define the age-gain of $Q_{i,\mathcal{S}}$ as $\delta_{i,\mathcal{S}} = h_i(k) - w_{i,\mathcal{S}}(k)$.

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Let $x_i(k)$ be the throughput debt associate with node i: $x_i(k+1) = kq_i - \sum_{\tau=1}^{k} d_i^{\pi}(\tau)$.

Define the rate-gain of user i as $f_i(k) = \left(\left(x_i(k) + q_i\right)^+\right)^2 - \left(\left(x_i(k) + q_i - 1\right)^+\right)^2$.

Scheduling Coding Actions

ARM policy: In each slot k, the ARM policy chooses the action that has the maximum weight

$A(k)$	**Weights**
$Q_{i,\varnothing}$	$(1-\epsilon_i)\Big(\beta_i\delta_{i,\varnothing}(k)+\lambda f_i(k)\Big)$
$\oplus_{u\in[l]}Q_{\tau_u,s_{\tau_u}}$	$\sum_{u=1}^{l}\beta_{\tau_u}\delta_{\tau_u,s_{\tau_u}}(k)(1-\epsilon_{\tau_u})$ $+\lambda\sum_{u=1}^{l}(1-\epsilon_{\tau_u})f_{\tau_u}(k)$

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Remark: We have observed in simulations that a good approximation of the above ARM policy is obtained by choosing the maximum clique size to be 2. This captures most of the gain with a much reduced complexity. The number of coding actions reduces from 2^{M^2} to $2M(M-1)^3$.

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Let $C^{uncoded}$ be the set of all tuples $\underline{q} = (q, q, q)$ for which the system is strongly stabilized. Then, define a symmetric stationary randomized policy:

$\Pr\{A(k) = Q_{i,\emptyset}\} = \mu_{i,\emptyset}$

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By symmetry, let $\mu_{i,\emptyset} = \mu$, $\mu_{\tau_1,\{\tau_2\},\tau_2,\{\tau_1\}} = \xi_1$, $\mu_{\tau_1,\{\tau_2\},\tau_2,\{\tau_1,\tau_3\}} = \xi_2$, $\mu_{\tau_1,\{\tau_2,\tau_3\},\tau_2,\{\tau_1,\tau_3\}} = \xi_3$, and $\mu_{\tau_1,\{\tau_2,\tau_3\},\tau_2,\{\tau_1,\tau_3\},\tau_3,\{\tau_1,\tau_2\}} = \xi_4$

Upper bound

$$\min_{\mu} \quad \frac{\frac{1}{3}\sum_{i=1}^{3}\alpha_i}{\theta} + \frac{\frac{1}{3}\sum_{i=1}^{3}\alpha_i}{\mu(1-\epsilon)} + \lambda$$

$$\text{s.t. } 3\mu + 3\xi_1 + 6\xi_2 + 3\xi_3 + \xi_4 = 1$$

$$\mu(1-\epsilon^3) \geq q$$

$$(\mu + \xi_2)(1-\epsilon^2) + \xi_1(1-\epsilon) \geq q$$

$$\mu(1 - 2\epsilon^2 + \epsilon^3) + 2\xi_1(1-\epsilon) + 2\xi_2(1-\epsilon^2) \geq q$$

$$(\mu + 2\xi_1 + 4\xi_2 + 2\xi_3 + \xi_4)(1-\epsilon) \geq q$$

$$\mu \geq 0, \xi_j \geq 0, j = 1,2,3,4$$

Tradeoff !

Lower bound

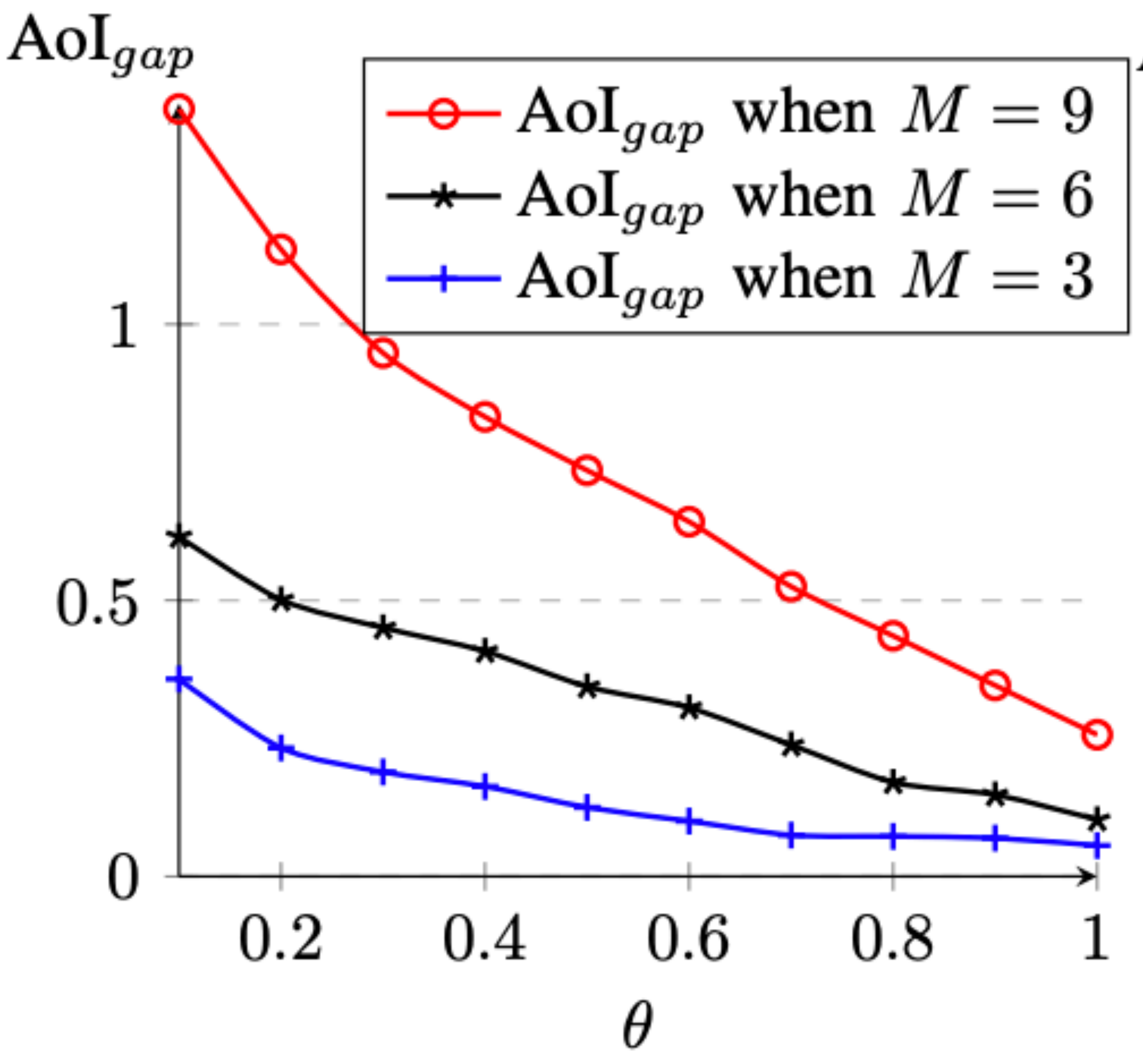
For any policy π with communication rate r_i^π, we have the following lower bound on $J^\pi(\underline{q})$:

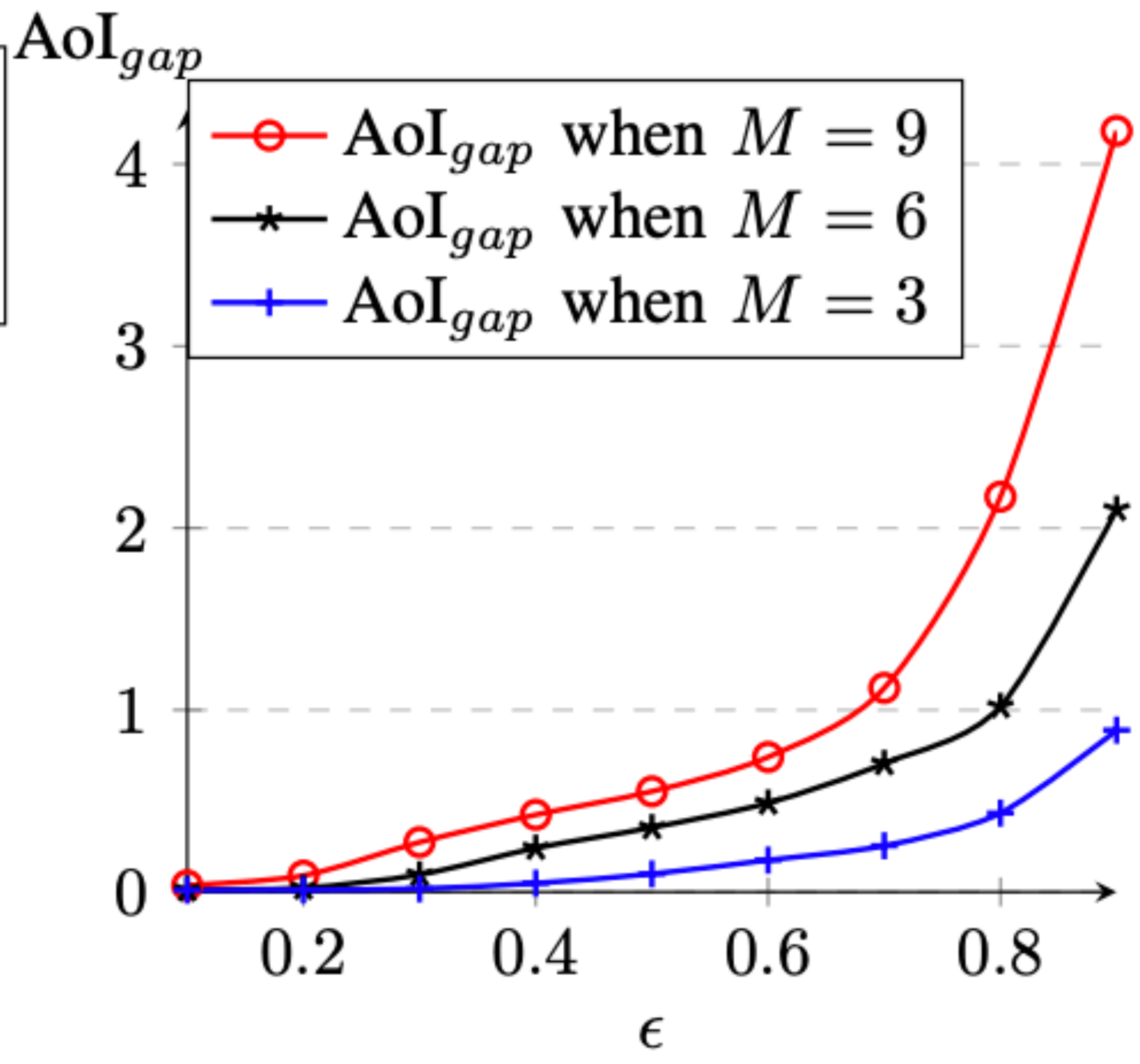
$$J^\pi(\underline{q}) \geq \frac{M}{2\sum_{i=1}^{M} r_i^\pi/\alpha_i} + \sum_{i=1}^{M} \frac{\alpha_i}{2M}$$

$$J^\pi(\underline{q}) \geq \frac{1}{M}\sum_{i=1}^{M} \frac{\alpha_i}{\theta_i}$$

Simulation Results

Benefits of Coding





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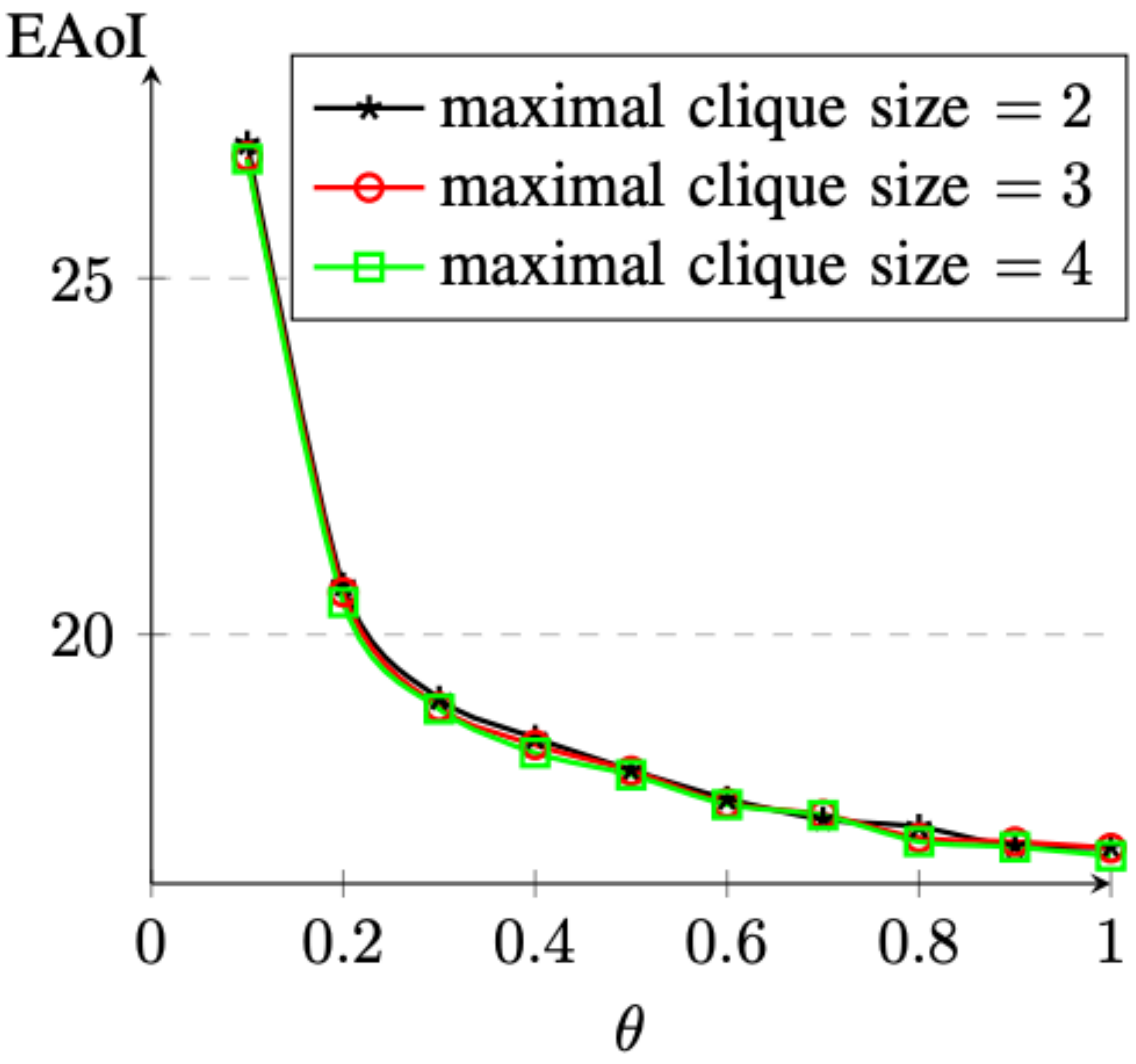
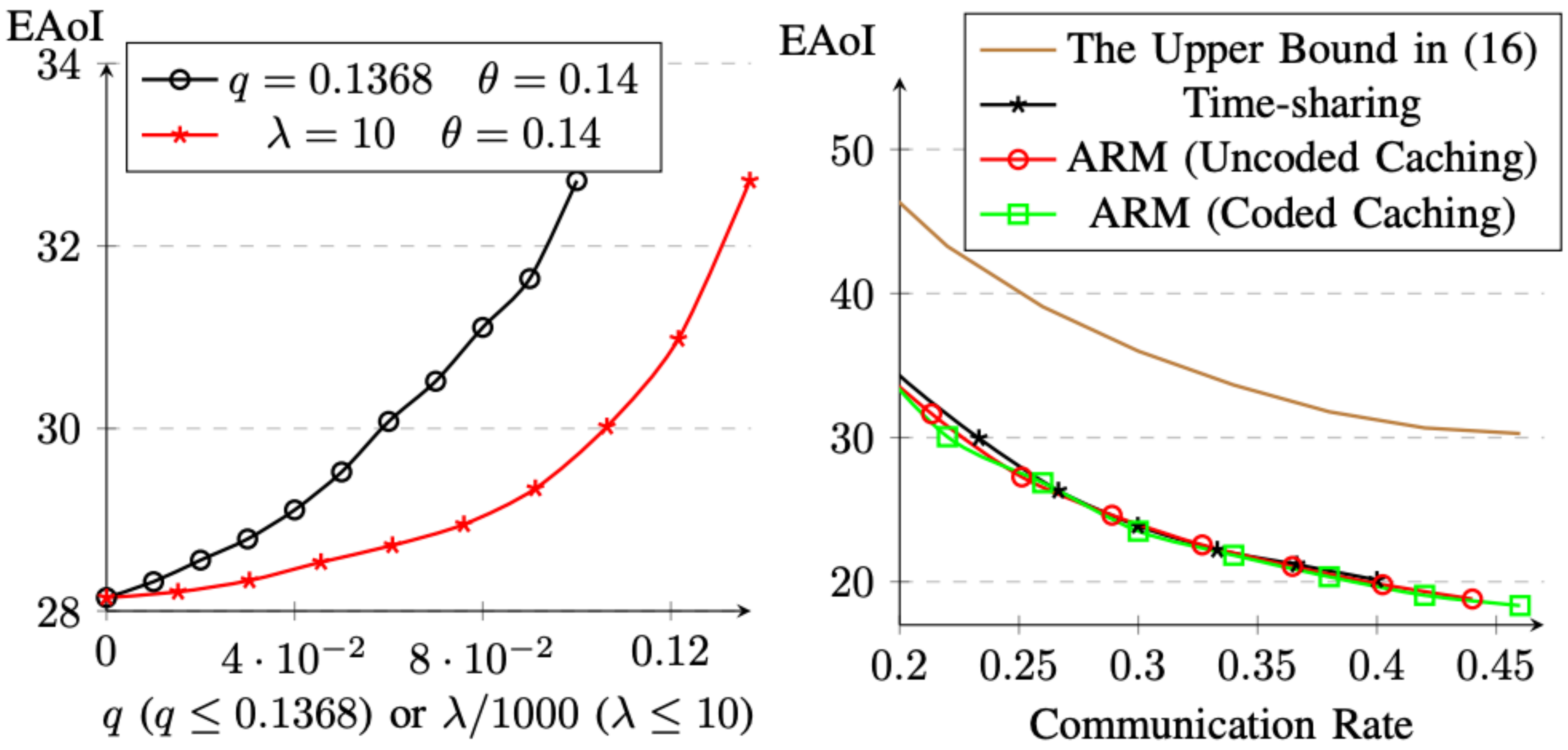
Impact of Maximal Clique Size



Simulation Results

Tradeoff between Age and Rate



(a) EAoI vs. q or λ.

(b) EAoI vs. rate.