Benefits of Coding on Age of Information in Broadcast Networks

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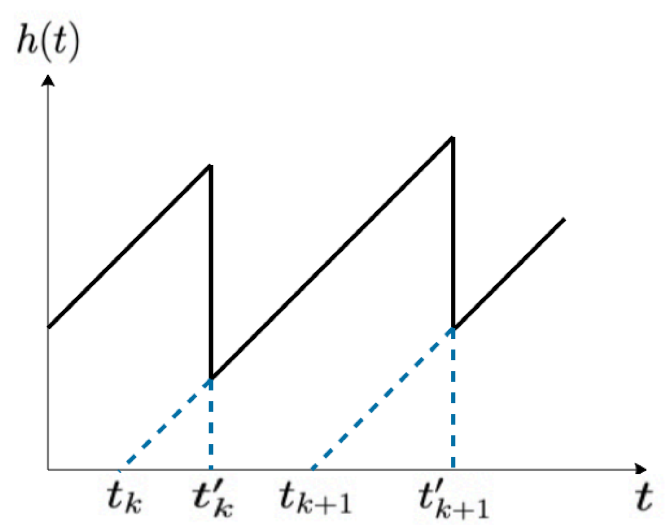
Age of Information

- t_k: The generation time of kth packet.
- t'_k: The receiving time of kth packet.
- $h(k)$: The age of destination node at time t.
- The-Average Age $= \lim_{T\to\infty} \frac{1}{T}\int_0^T h(t)dt$.



Motivation

- From [1], coding not useful in point to point erasure channels.
- Monitoring two source through a broadcast packet erasure channel (BPEC).
- Is coding beneficial (in terms of AoI)?

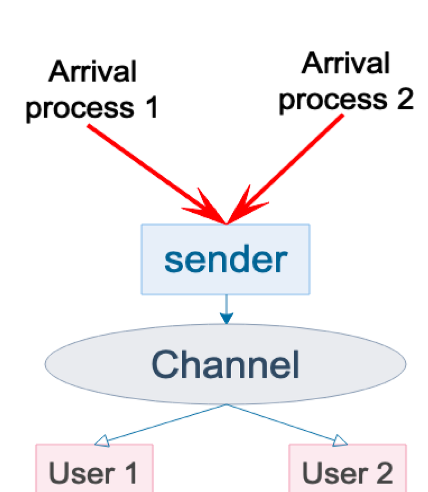
Model

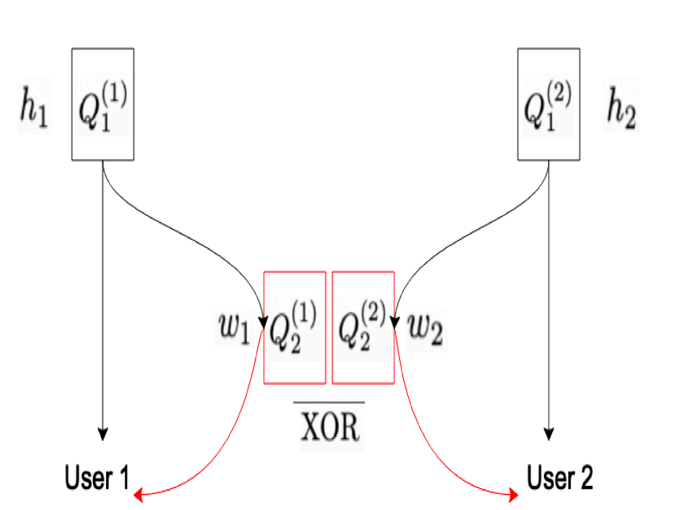
- Transmission occurs on a BPEC.
- Discrete-time model, packet arrival per slot, deterministic service time of one slot, network management
- Expected Weighted Sum AoI (EWSAoI)

$$\text{EWSAoI} = \mathbb{E}[\frac{1}{2T}\sum_{t=1}^{T}\sum_{i=1}^{2}\alpha_i h_i(t)]$$

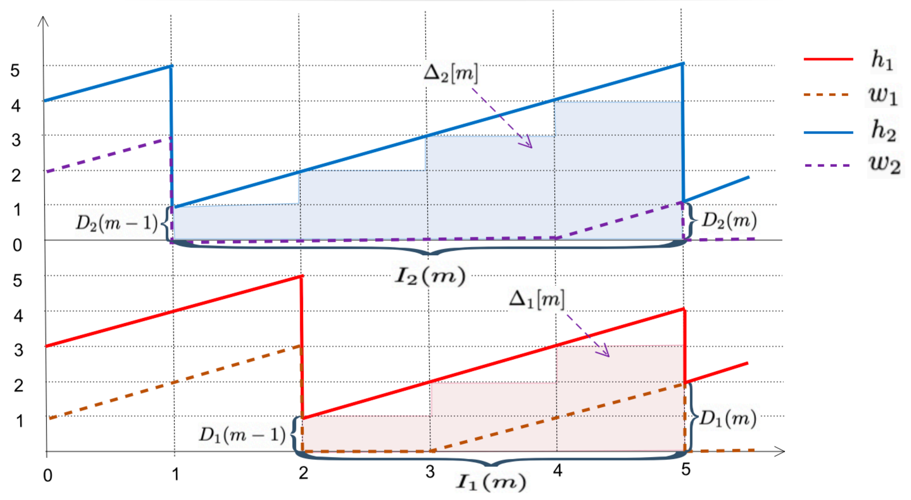
- Goal: Minimizing the EWSAoI, devising near-optimal policies
- Encoder actions:

$$A(k) = \begin{cases} 1 & \text{a packet is transmitted from } Q_1^{(1)} \\ 2 & \text{a packet is transmitted from } Q_1^{(2)} \\ 3 & \text{a coded packet is transmitted from the XOR} \end{cases}$$





A Sample Path



k	1	2	3	4	5
$A(k)$	2	1	1	2	3
$(Z_1(k), Z_2(k))$	(1,1)	(1,0)	(0,0)	(0,0)	(1,1)

A Lower Bound

- $N_i(T)$: the total number of packets received by user i up to T.
- $J_T^\pi \geq \frac{1}{4}\sum_{i=1}^2 \alpha_i \frac{T}{N_i(T)} + \frac{1}{4}$.
- The capacity of two-user any BPEC satisfies

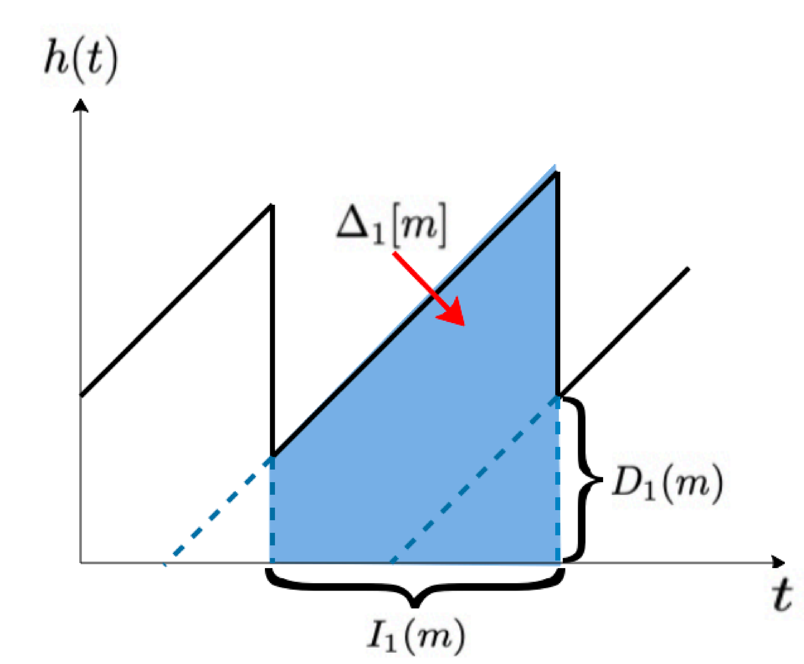
$$\lim_{T\to\infty} \frac{\frac{N_1(T)}{T}}{1-\epsilon_1} + \frac{\frac{N_2(T)}{T}}{1-\epsilon_{12}} \leq 1, \quad \lim_{T\to\infty} \frac{\frac{N_1(T)}{T}}{1-\epsilon_{12}} + \frac{\frac{N_2(T)}{T}}{1-\epsilon_2} \leq 1.$$

- Using Cauchy-Schwarz Inequality to get the following result.

> For any communication policy π, we have:
>
> $$\mathbb{E}[J^\pi] \geq \frac{1}{4}\left(\frac{(\sum_{i=1}^2 \sqrt{\alpha_i(2-\epsilon_{12}-\epsilon_{\setminus i})})^2}{(1-\epsilon_{12})(2-\epsilon_1-\epsilon_2)} + 1\right).$$
>
> where $\alpha_1 + \alpha_2 = 1$, $0 \leq \alpha_1, \alpha_2 \leq 1$.

Randomized Policies

Some fixed probability vector (μ_1, μ_2, μ_3). Consider user 1,



By renewal process,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{k=1}^{T}\mathbb{E}[h_1(k)|\underline{s}(1)] = \lim_{T\to\infty}\frac{1}{T}\sum_{m=1}^{N_1(T)}\mathbb{E}[\Delta_1(m)|\underline{s}(1)]$$
$$= \lim_{T\to\infty}\frac{N_1(T)}{T}\left(\frac{\mathbb{E}[I_1^2]}{2} + \mathbb{E}[D_1]\mathbb{E}[I_1] - \frac{\mathbb{E}[I_1]}{2}\right)$$
$$= \frac{\mathbb{E}[I_1^2]}{2\mathbb{E}[I_1]} + \mathbb{E}[D_1] - \frac{1}{2}.$$

EWSAoI of Randomized policy

The EWSAoI of Randomized policy is characterized by

$$\mathbb{E}[J^R] = \frac{1}{2}\sum_{i=1}^{2}\alpha_i\left(\frac{\frac{\mu_3(1-\epsilon_i)}{(\mu_i(1-\epsilon_{12}))^2} + \frac{-\mu_i(\epsilon_i-\epsilon_{12})}{((\mu_i+\mu_3)(1-\epsilon_i))^2}}{\frac{\mu_3(1-\epsilon_i)}{\mu_i(1-\epsilon_{12})} + \frac{-\mu_i(\epsilon_i-\epsilon_{12})}{(\mu_i+\mu_3)(1-\epsilon_i)}} + \frac{\mu_3(\epsilon_i-\epsilon_{12})}{(\mu_i+\mu_3)(1-\epsilon_{12})(\mu_i(1-\epsilon_{12})+\mu_3(1-\epsilon_i))}\right).$$

Consider a symmetric case:

- $\epsilon_1 = \epsilon_2 = \epsilon$, $\epsilon_{12} = \epsilon_{12}(\epsilon)$.
- $P(A(k)=1) = P(A(k)=2) = \mu$, $P(A(k)=3) = 1-2\mu$.

> The optimal μ^* is characterized in
>
> $$\mu^* = \begin{cases} \frac{\sqrt{1-\epsilon}}{\sqrt{1-\epsilon}+\sqrt{\epsilon-\epsilon_{12}(\epsilon)}} & \epsilon_{12}(\epsilon) - 2\epsilon + 1 < 0 \\ 1/2 & \epsilon_{12}(\epsilon) - 2\epsilon + 1 \geq 0. \end{cases}$$

> *Optimal coded and uncoded randomized policies achieve the same EWSAoI if and only if $\epsilon_{12}(\epsilon) - 2\epsilon + 1 \geq 0$. Otherwise, optimal coded randomized policies strictly outperform uncoded randomized policies.*

Max-Weight Policies

- Define the Lyapunov function

$$L(\underline{h}(k)) = \frac{1}{2}\sum_{i=1}^{2}\alpha_i h_i^2(k).$$

- Define the one-slot Lyapunov Drift

$$\Theta(\underline{h}(k)) = \mathbb{E}[L(\underline{h}(k+1)) - L(\underline{h}(k))|\underline{s}(k)].$$

- Max-Weight policies minimize the one-slot Lyapunov drift.

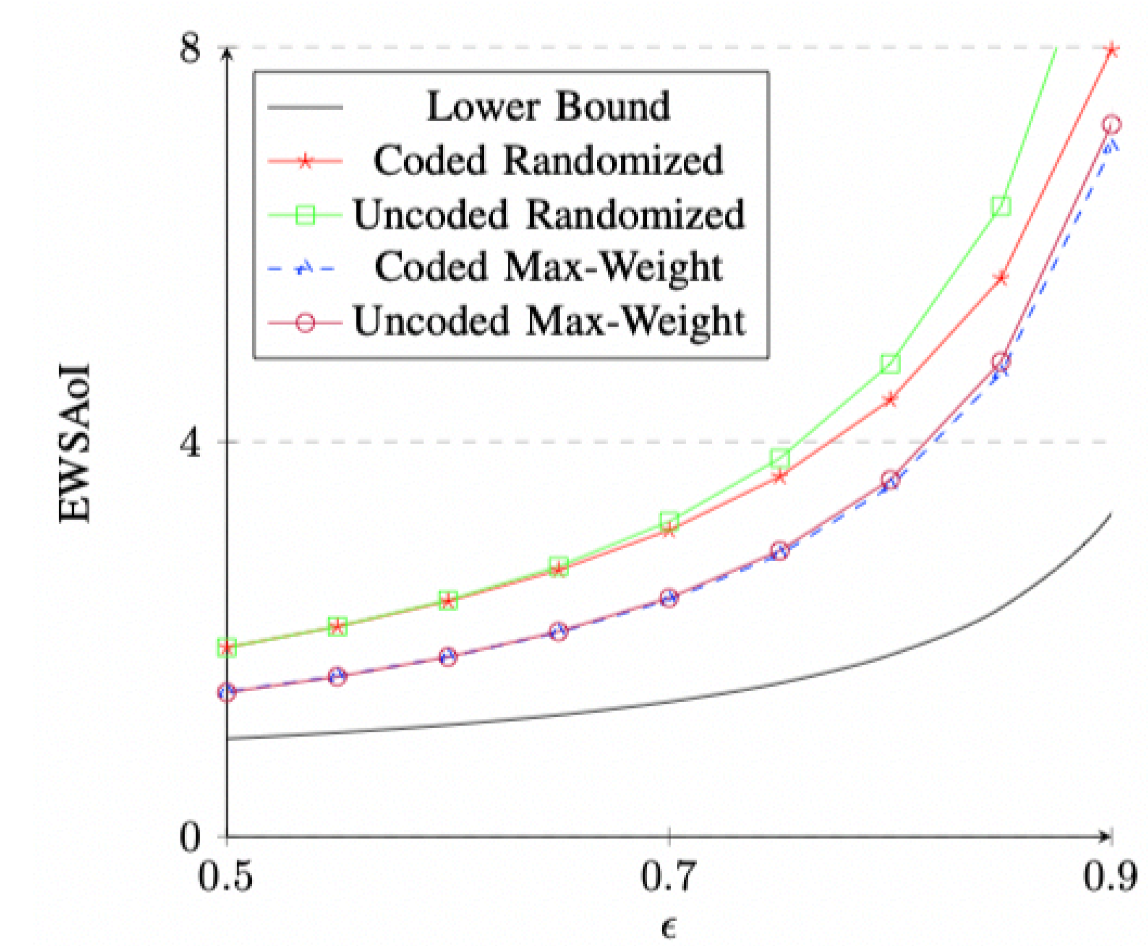
Max-Weight Policy

In each slot k, the Max-Weight policy chooses the action that has the maximum weight as shown in following Table:

$A(k)$	Weights
1	$\frac{\alpha_1(1-\epsilon_1)}{2}h_1(k)(h_1(k)+2)$
2	$\frac{\alpha_2(1-\epsilon_2)}{2}h_2(k)(h_2(k)+2)$
3	$\frac{1}{2}\sum_i \alpha_i(1-\epsilon_i)1_{\{w_i(k)>0\}}(h_i^2(k)+2h_i(k)-w_i^2(k)-2w_i(k))$

Numerical Results

- $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, $\epsilon_{12} = \epsilon^2/5$, $\epsilon \in [0.5, 0.9]$.
- Coding is beneficial when the channel erasure is larger than a threshold.



References

[1] E. Najm, E. Telatar, and R. Nasser "Optimal age over erasure channels", arXiv: 1901.01573, 2019.

[2] I. Kadota, A. Shiha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Scheduling policies for minimizing age of information in broadcast wireless networks', arXiv: 1801.01803, 2018.

[3] X. Chen, S. Saeedi Bidokhti, "Benefits of Coding on Age of Information in Broadcast Networks," in *2019 IEEE Information Theory Workshop*, 2019 [Accepted].

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