Age of Information in Random Access Channels

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Abstract—In applications of remote sensing, estimation, and control, timely communication is critical but not always ensured by high-rate communication. This work proposes decentralized age-efficient transmission policies for random access channels with M transmitters. We propose the notion of *age-gain* of a packet to quantify how much the packet will reduce the instantaneous age of information at the receiver side upon successful delivery. We then utilize this notion to propose a transmission policy in which transmitters act in a decentralized manner based on the age-gain of their available packets. In particular, each transmitter sends its latest packet only if its corresponding age-gain is beyond a certain threshold which could be computed adaptively using the collision feedback or found as a fixed value analytically in advance. Both methods improve age of information significantly compared to the state of the art. In the limit of large M, we prove that when the arrival rate is small (below $\frac{1}{eM}$), slotted ALOHA-type algorithms are order optimal. As the arrival rate increases beyond $\frac{1}{eM}$, while age increases under slotted ALOHA, it decreases significantly under the proposed age-based policies. For arrival rates θ, $\theta = \frac{1}{o(M)}$, the proposed algorithms provide a multiplicative gain of at least two compared to the minimum age under slotted ALOHA (minimum over all arrival rates). We conclude that it is beneficial to increase the sampling rate (and hence the arrival rate) and transmit packets selectively based on their age-gain. This is surprising and contrary to common practice where the arrival rate is optimized to attain the minimum AoI. We further extend our results to other random access technologies such as Carrier-sense multiple access (CSMA).

Index Terms—Age of information, random access, collision channel, distributed algorithms, stochastic arrival, slotted ALOHA, carrier sensing multiple access.

I. INTRODUCTION

COMMUNICATION networks have witnessed a rapid growth in the past few decades and they have laid a path to the integration of intelligence into cyber-physical systems,

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the Internet of Things, smart cities, as well as healthcare systems. Today, state-of-the-art network communication strategies are considered reliable and high speed; nevertheless, they often do not perform satisfactorily for time-sensitive applications. For example, in applications of remote sensing, estimation, and control, high-rate communication does not ensure timely communication of data. As a matter of fact, it is often observed that as the capacity of a system is approached, the delay increases significantly and hence so does the age of information.

Age of information (AoI), introduced in [1], [2], measures the freshness of information at the receiver side. AoI is a function of both how often packets are transmitted and how much delay packets experience in the communication network. When the rate of communication is low, the receiver's AoI will increase (implying that the receiver's information is stale) because the transmitter is not sending packets frequently enough. But even when the transmitter is sending packets frequently, if the system design imposes a large delay for the packets, the information at the receiver will still be stale. The metric of AoI is of great importance in the Internet of Things applications where timeliness of information is crucial (e.g. in monitoring the status of a system). Another interesting application domain of AoI is in communication for estimation and control [3], [4] where estimation error increases (exponentially) by time before new packets (samples) are received at the destination. It is believed that minimizing AoI is a good proxy for minimizing estimation error [4]–[6].

Assuming a first come first serve (FCFS) policy, the work in [7], [8] show in queue theoretic setups that AoI is minimized at an optimal update rate. Relaxing the restriction of FCFS policies, [8], [9] propose packet management policies that discard old packets and improve AoI in wide regimes of operation. This already points to the fact that, under the metric of AoI, rate and reliability have little relevance in the design of timely communication schemes. This is because AoI implicitly assumes that the information content of the packets form a Markov process and hence fresh packets render older packets obsolete. In the past few years, various extensions and new dimensions have also been studied in the paradigm of timely communication: source and channel coding were studied in [10]–[13], multi-hop networks were studied in [14]–[16], and scheduling algorithms were studied in [17]–[23].

This paper considers the problem of minimizing age of information over a random access channel. This setup is particularly relevant in remote estimation and control of processes that are observed from decentralized sensors in

TABLE I

SUMMARY OF THE PROPOSED ALGORITHMS AND BOUNDS

Algorithms or Bounds	Key ideas	Normalized age performance ($M \to \infty$)
Proposition 1	There is always a fresh packet to be transmitted	Lower bound $\frac{1}{2C_{RA}}$; C_{RA} is the capacity of the RA channel
Proposition 2	All packets are delivered upon arrival	Lower bound $\frac{1}{\theta M}$; tight when $\theta < \frac{1}{eM}$
Slotted ALOHA	See details in [36, Chapter 4.2.3]	Normalized age $\frac{1}{\theta M}$ when $\theta < \frac{1}{eM}$
Algorithm 1	Adaptive age-based thinning (ALOHA)	Decreases age and increases throughput simultaneously
Algorithm 2	Stationary age-based thinning (ALOHA)	Normalized age $\frac{e}{2}$ for $\theta = \frac{1}{o(M)}$
Algorithm 3	Stationary age-based thinning (RA with maximum throughput C)	Normalized age $\frac{1}{2C}$ for $\theta = \frac{1}{o(M)}$

wireless networks (see also our follow up work [6]). Prior work such as [17], [18], [22], [24] consider scheduling policies in multiple access channels that are controlled in a centralized manner. However, in decentralized (random access) applications, employing such policies would require a huge amount of communication and coordination rendering them inapplicable. Towards designing *decentralized* algorithms for minimizing age of information, [25], [26] analyze stationary randomized policies under the assumption that sources generate packets in every time slot (i.e., all sources are active at all times). Considering the more realistic scenario where packets are generated at random times, [27] analyzes round-robin scheduling techniques with and without packet management and also presents partial results for stationary randomized policies. Round-robin policies are proved to be age-optimal in [28] when the number of transmitters are large and the arrival rate is constant. The followup work [29] additionally assumes that nodes are provided with carrier sensing capabilities and proposes distributed schemes that have good performance in simulations; Nevertheless, [29] does not address how the parameters of the proposed algorithms should be designed theoretically. The concurrent work [30], [31] (published after our work [32]) investigate variants of decentralized age-based schemes for CSMA under energy constraints. In an unslotted, uncoordinated, unreliable multiple access collision channel, [33] provides the exact system age and an accurate individual age approximation for a small number of sources. The work [34] which was done independently and concurrently to this work considers a threshold-based lazy version of Slotted ALOHA where each transmitter attempts to access the channel with a certain probability when its corresponding age exceeds a certain threshold. Optimizing the threshold and the transmission probabilities are non-trivial and the authors provide analysis only for $M = 2$ transmitters for the special case where the arrival rate is equal to 1.

In this work, we design decentralized age-based transmission policies and provide upper and lower bounds on the achievable AoI in interesting regimes of operation. The major part of this paper deals with random access technologies such as slotted ALOHA that do not assume carrier sensing capabilities. The underlying reason is threefold: (i) Status packets are generally very short (as opposed to traditional settings such as streaming where packets are long) and so CSMA is not efficient; (ii) Transmitters have low power capabilities. As such, it is not very efficient (in terms of energy and cost) to perform carrier sensing when the arrival rate is large and CSMA is not particularly useful when the arrival

rate is small. More importantly, since transmission power is low, the hidden node problem will be a major issue under CSMA-type protocols; (iii) Our analytical results are clearer without the additional complexity of CSMA. In Section V-E, we describe how our findings generalize and apply to CSMA.

The contributions of this paper are as follows. In presenting our results below, we assume large symmetric networks in which we have M transmitters and each transmitter has arrival rate θ. The key ideas are summarized in Table I.

- We first derive two general lower bounds on AoI for any transmission policy by considering two ideal cases: (i) there is always a fresh packet to be transmitted and hence delivered packets are assumed to experience minimum delay; (ii) all packets are delivered instantaneously upon their arrivals with minimum delay, but without experiencing collisions. The former lower bound turns out to be active as the arrival rate (θ) approaches 1, and the latter lower bound becomes active when θ is small, i.e., when the inter-arrival time is the dominant term of the inter-delivery time.

- We analyze the well-known slotted ALOHA algorithm. It is known that slotted ALOHA is stable when the sum arrival rate is below the infamous critical point $\frac{1}{e}$. But it becomes unstable when the sum arrival rate is larger than $\frac{1}{e}$. We prove that when the sum arrival rate is below $\frac{1}{e}$, the normalized age performance of a (stabilized) slotted ALOHA algorithm, properly defined later, is approximate $\frac{1}{M\theta}$ in the limit of large M and is optimal. We further show numerically that the normalized age performance is close to that of centralized max-weight policies that schedule based on *age-gain* (which is formally defined in Section IV) when the sum arrival rate is less than $\frac{1}{e}$. Simulation results show that as the sum arrival rate increases beyond this critical point, the normalized age of slotted ALOHA explodes.

- We then ask if we can reduce age as the sum arrival rate increases beyond the critical point $\frac{1}{e}$. This is an important question that sheds light on whether increasing the sampling rate is useful when communication is over a random access channel. We find an affirmative answer. We propose two age-based thinning algorithms, i.e., Algorithm 1 and Algorithm 2. The core idea for both algorithms is that transmitters selectively disregard packets in order to mimic an effective (sum) arrival rate equal to $\frac{1}{e}$. In particular, we develop a threshold policy that can be implemented in a decentralized

manner at the transmitters and in which packets that offer large age-gains are transmitted and those that offer small age-gains are disregarded. In Algorithm 1 we propose an adaptive threshold in which the threshold is updated and improved based on the channel feedback. Algorithm 2 proposes a stationary threshold, in which the threshold is predetermined and thus saves computation costs. Using Algorithm 2, i.e., the stationary thinning method, we prove asymptotically ($M \rightarrow \infty$) that for any θ that is not too small $\left(\theta = \frac{1}{o(M)}\right)$, the normalized age is approximate $\frac{e}{2}$ and twice better than that the minimum age that (stabilized) slotted ALOHA can attain. Furthermore, numerical results show that as θ approaches 1, the normalized age approaches 1 using Algorithm 1 (the adaptive thinning method) that adaptively optimizes the threshold in each time slot. Interestingly, we observe that the adaptive thinning algorithm attains a smaller age while increasing the throughput beyond what slotted ALOHA can achieve.

- Finally, we generalize our stationary thinning mechanism (Algorithm 2), and demonstrate that the idea behind Algorithm 2 is useful for other random access technologies (e.g. CSMA), see Algorithm 3. In particular, we prove that given a technology that can achieve the throughput C (without coding), Algorithm 3 can attain the normalized age of $\frac{1}{2C}$. Numerical results show that it approaches order-optimality in the limit of large M.

The rest of the paper is organized as follows. Section II introduces the system model and notations. Section III provides lower bounds on NAAoI and Section IV proposes centralized Max-Weight scheduling policies to avoid collisions and ensure small NAAoI. Section V introduces novel decentralized age-based policies and provides asymptotic analysis of their corresponding NAAoI (as $M \rightarrow \infty$). In Section VI, we numerically compare the achievable age of the proposed distributed transmission policies with centralized policies as well as the derived lower bounds and demonstrate that our asymptotic results hold approximately for moderate values of M as well. We finally conclude in Section VII and discuss future research directions.

II. SYSTEM MODEL AND NOTATION

We consider a wireless architecture where a controller monitors the status of M identical source nodes over a shared wireless medium. To provide analytical frameworks and closed form solutions, we focus on the symmetric systems (instead of asymmetric ones), and use the profile of all sources as an estimate on an individual source and look at the limit behaviour. Let time be slotted. At the beginning of every slot k, $k = 1, 2, \ldots$, the source node i, $i = 1, \ldots, M$, generates a new packet encoding information about its current status with probability θ and this packet becomes available at the transmitter immediately. We denote this generation/arrival process at the transmitter by $A_i(k)$, where $A_i(k) = 1$ indicates that a new packet is generated at time slot k and $A_i(k) = 0$ corresponds to the event where there is no new update. New packets are assumed to replace undelivered older packets at the

source (i.e., older packets are discarded), relying on the fact that the underlying processes that are monitored in physical systems are oftentimes Markovian.[1]

The communication media is modeled by a collision channel: If two or more source nodes transmit at the beginning of the same slot, then the packets interfere with each other (collide) and do not get delivered at the receiver. We use the binary variable $d_i(k)$ to indicate whether a packet is transmitted from source i and received at the destination in time slot k. Specifically, $d_i(k) = 0$ if source i does not transmit at the beginning of time slot k or if collision occurs; $d_i(k) = 1$ otherwise.

We assume a delay of one time unit in the delivery of packets, meaning that packets are transmitted at the beginning of time slots and, if there is no collision, they are delivered at the end of the same time slot. We assume that all transmitters are provided with channel collision feedback at the end of each time slot. Specifically, at the end of time slot k, $c(k) = 1$ if collision happened and $c(k) = 0$ otherwise. In the event that collision occurs, the involved transmitters can keep the undelivered packets and retransmit them according to their transmission policy (until the packets are successfully delivered or replaced by new packets).

Our objective is to design *decentralized* transmission mechanisms to minimize time-average age of information per source node. A decentralized transmission policy is one in which the decision of transmitter i at time k is dependent only on its own history of actions, the packets arrived so far, $\{A_i(j)\}_{j=1}^{k}$, as well as the collision feedback received so far, $\{c(j)\}_{j=1}^{k-1}$.

The measure of performance in this work is Age of Information (AoI). Originally defined in [1], [2], AoI captures the timeliness of information at the receiver side. We extend the definition a bit further, formally defined below, to also account for the age of information at the source side. Aging at the source/transmitter is caused by the *stochastic nature of arrivals*.

Definition 1: Consider a source-destination pair. Let $\{k_\ell\}_{\ell \geq 1}$ be the sequence of generation times of packets and $\{k'_\ell\}_{\ell \geq 1}$ be the sequence of times at which those packets are received at the destination. At any time τ, denote the index of the last generated packet by $n_s(\tau) = \max\{\ell | k_\ell \leq \tau\}$ and the index of the last received packet by $n_d(\tau) = \max\{\ell | k'_\ell \leq \tau\}$. The *source's age of information* is defined by $w(k) = k - k_{n_s(k)}$ and the *destination's age of information* is defined by $h(k) = k - k_{n_d(k)}$.

It is clear from the above definition that once there is a new packet available at the transmitter, the older packet(s) cannot contribute to reducing the age of the system. We hence assume without loss of generality that buffers at transmitters are of size 1 and new packets replace old packets upon arrival. We formalize and prove this claim in Appendix A.

Following Definition 1, let $h_i(k)$ denote the destination's AoI at time slot k with respect to source i. The age $h_i(k)$ increases linearly as a function of k when there is no packet

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[1]We show in Appendix A that this assumption can be made without loss of generality when the performance measure is Age of Information.

delivery from source i and it drops with every delivery to a value that represents how old the received packet is; within our framework, this would be the corresponding source's AoI (in previous time slot) plus 1. Without loss of generality, we assume $w_i(1) = 0$ and $h_i(1) \geq 0$, and write the recursion of AoI as follows:

$$h_i(k) = \begin{cases} w_i(k-1)+1 & d_i(k-1)=1 \\ h_i(k-1)+1 & d_i(k-1)=0 \end{cases} \quad (1)$$

and

$$w_i(k) = \begin{cases} 0 & A_i(k)=1 \\ w_i(k-1)+1 & A_i(k)=0. \end{cases} \quad (2)$$

Note that at the beginning of each time slot k, given the collision feedback $\{c(j)\}_{j \leq k-1}$ and local information about $\{A_i(j)\}_{j \leq k}$, transmitter i can compute its corresponding source's AoI $\{w_i(j)\}_{j \leq k}$ and destination's AoI $\{h_i(j)\}_{j \leq k}$.

We define the Normalized Average AoI (NAAoI) as our performance metric of choice:[2]

$$J^\pi(M) = \lim_{K \to \infty} \mathbb{E}[J_K^\pi], \quad J_K^\pi = \frac{1}{MK} \sum_{i=1}^{M} \sum_{k=1}^{K} \frac{h_i^\pi(k)}{M} \quad (3)$$

where π refers to the underlying transmission policy.

We consider *centralized policies* and *decentralized age-based policies* in this work. Centralized policies serve as benchmarks. They need a central scheduler who receives information about all arrival processes and previous transmission actions, and coordinate all the transmitters. When the number of transmitters M gets large, facilitating such scales of coordination is not feasible and we are hence interested in decentralized mechanisms. Randomized policies are easy to implement in a decentralized manner. Previous works [25], [26] fall into this class but they have the weakness of not utilizing local collision feedback at the transmitters. Utilizing the collision feedback, we aim to make age-based decisions at the transmitters in a decentralized manner.

A. Notation

We use the notations $\mathbb{E}[\cdot]$ and $\Pr(\cdot)$ for expectation and probability, respectively. We denote scalars with lower case letters, e.g. s; vectors with underlined lowercase letters, e.g. $\underline{s}$, and matrices with boldface capital letters, e.g. $\mathbf{S}$. Notation $[\underline{s}]_i$ represents the ith element of $\underline{s}$ and $[\mathbf{S}]_{ij}$ denotes the element in the ith row and j^{th} column. Random variables are denoted by capital letters, e.g. S. We use M to denote the number of transmitters, K to denote the time horizon, and C to denote the capacity of a channel (under a given technology). The operator $(s)^+$ returns 0 if $s < 0$ and it returns s if $s \geq 0$. $\lfloor s \rfloor$ represents the largest integer j such that $j \leq s$. $O(\cdot)$ and $o(\cdot)$ represent the Big O and little o notations according to Bachmann-Landau notation, respectively. We summarize the notations in Table II.

[2]For any distributed transmission scheme, it is clear that the average AoI increases with the number of source node M for any fixed arrival rate θ. Note that our problem setup allows M to become very large, so to offset the effect introduced by the number of source nodes, we consider the proposed NAAoI.

TABLE II

USEFUL NOTATIONS

| M	The number of sources
K	The time horizon
θ	The generation/arrival rate of new packets
$A_i(k)$	The indicator of the generation/arrival process
$d_i(k)$	The indicator of delivery at source i
$\lambda_i(k)$	The indicator of transmission at source i
$c(k)$	The indicator of collision in the channel
$h_i(k)$	The destination's AoI at time k w.r.t source i
$w_i(k)$	The source's AoI at time k w.r.t source i
π	A specific transmission and sampling policy
$J^\pi(M)$	Normalized Average AoI with M sources
C_{RA}	The sum-capacity of the random access channel
$\delta_i(k)$	The age-gain in time slot k at source i
$\{\ell_m(k)\}_m$	The distribution of age-gain in time slot k
$\mathtt{T}(k)$	The threshold under the AAT policy at time k
$\mathtt{T}^*$	The threshold under the SAT policy
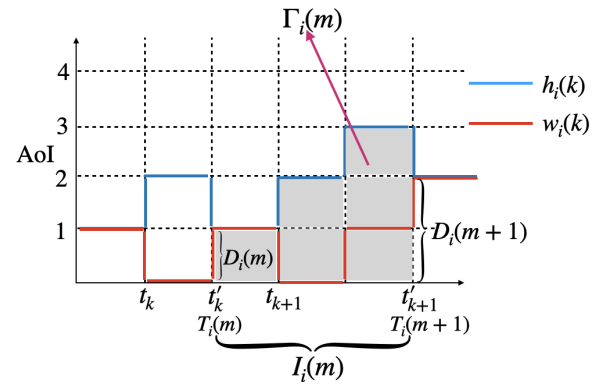
Fig. 1. an example of $D_i(m)$, $I_i(m)$, and $\Gamma_i(m)$.

III. LOWER BOUND

We start by deriving two lower bounds on the achievable age performance. The first lower bound is derived by assuming that there is always a fresh packet to be transmitted (and hence delivered packets are assumed to experience unit-time delays). The second lower bound is derived by assuming that all packets are delivered instantaneously upon their arrivals (with unit-time delays, but without experiencing collisions). The former is active as θ approaches 1 and the latter is active when θ is small (when the inter-arrival time is the dominant term of the inter-delivery time).

Fix a large time horizon K and look at the packets of source i. Let $N_i(K)$ denote the number of delivered packets (from source i) up to and including time slot K. Now consider the mth and $(m+1)^{th}$ *deliveries* at the receiver and denote the delivery time of them at the receiver by $T_i(m)$ and $T_i(m+1)$, respectively. The inter-delivery time

$$I_i(m) = T_i(m+1) - T_i(m)$$

is the time between these two consecutive deliveries. Upon arrival of the mth delivered packet at the receiver, the age of information at the receiver drops to the value $D_i(m)$ which represents how much delay the packet has experienced in the system. Fig. 1 illustrates the introduced notation. Now define $\Gamma_i(m)$ as the sum of age functions $h_i(k)$, where k is in the

interval $[T_i(m), T_i(m+1))$:

$$
\begin{aligned}
\Gamma_i(m) &= \sum_{k=T_i(m)}^{T_i(m)+I_i(m)-1} h_i(k) \\
&= \frac{1}{2}\big(D_i(m) + I_i(m) + D_i(m)\big) \cdot I_i(m) \\
&\quad - \frac{I_i(m)}{2} \\
&= \frac{1}{2}I_i^2(m) - \frac{1}{2}I_i(m) + D_i(m)I_i(m).
\end{aligned}
\tag{4}
$$

It follows that in the limit of large K, we have

$$
J^\pi(M) = \lim_{K\to\infty} \mathbb{E}[J_K^\pi] = \lim_{K\to\infty} \mathbb{E}\left[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{m=1}^{N_i(K)}\Gamma_i(m)\right].
$$

Using this formulation, we next lower bound NAAoI. Let C_{RA} denote the sum-capacity of the underlying random access channel. Note that in the limit of large K, $\frac{N_i(K)}{K}$ is the throughput of transmitter i and

$$
\lim_{K\to\infty} \sum_{i=1}^{M} \frac{N_i(K)}{K} \le C_{RA}.
\tag{5}
$$

Then, we have the following propositions.

Proposition 1: For any transmission policy π,

$$
J^\pi(M) \ge \frac{1}{2C_{RA}} + \frac{1}{2M}.
$$

Proof: The proof is given in Appendix B. □

Proposition 2: For any transmission policy π,

$$
J^\pi(M) \ge \frac{1}{M\theta}.
\tag{6}
$$

Proof: The proof is given in Appendix C. □

Let us give an example of how Proposition 1 can be utilized. Note that C_{RA} is not known in general. Nevertheless, any upper bound on C_{RA} gives a lower bound on the normalized age. Based on [36], the capacity of the random access channel with collision feedback, in the limit of large M, is upper bounded by $\lim_{M\to\infty} C_{RA} \le 0.568$ and hence

$$
\lim_{M\to\infty} J^\pi(M) \ge .88.
\tag{7}
$$

Remark 1: The lower bound in (7) does not assume CSMA capabilities. For CSMA, we have $C_{CSMA} \le 1$ and hence

$$
J^\pi(M) \ge \frac{1}{2} + \frac{1}{2M}.
\tag{8}
$$

We show the asymptotic optimality of this bound in Section V-E as $M \to \infty$.

IV. CENTRALIZED SCHEDULING

The first class of schemes that we consider are centralized schemes that avoid collision by scheduling transmitters one by one. In particular, Max-Weight policies are shown to perform close to optimal in various works such as [17], [18], [24]. Although such schemes are not practical (due to the scale of required coordination), it turns out that they provide useful intuitions and they also serve as a benchmark for comparison in Section V. We assume a central scheduler that can observe

all arrival processes and coordinate/control all senders' actions in order to avoid collision.

Denote by $\lambda_i(k) = 1$ the event that transmitter i sends a packet and recall that $d_i(k)$ indicates delivery of packets. Note that if $\lambda_j(k) = 1$ for another source $j \ne i$, then the packets collide and no packets will be delivered. One can thus write

$$
d_i(k) = \lambda_i(k) \prod_{j \ne i} \big(1 - \lambda_j(k)\big).
\tag{9}
$$

The goal of a central scheduler is to select one source for transmission at each time. Denote $\underline{h}(k) = (h_1(k), h_2(k), \cdots, h_M(k))$. Following the works in [17], [18], [24], an age-based max-weight policy can be designed by considering the following Lyapunov function:

$$
\mathcal{L}(\underline{h}(k)) = \sum_{i=1}^{M} h_i(k)
\tag{10}
$$

and minimizing its corresponding one-step Lyapunov Drift:

$$
\Delta(\underline{h}(k)) = \mathcal{L}(\underline{h}(k+1)) - \mathcal{L}(\underline{h}(k)).
\tag{11}
$$

It turns out that the max-weight policy selects, in each time slot k, the transmitter that offers the highest *age-gain* $\delta_i(k)$, defined below:

$$
\delta_i(k) := h_i(k) - w_i(k).
\tag{12}
$$

$\delta_i(k)$ quantifies how much the instantaneous receiver's age of information reduces upon successful delivery from transmitter i. Proposition 3 states the above max-weight policy more formally (see Appendix D for the proof).

Proposition 3: For every time slot k, define

$$
\ell(k) = \arg\max_i \delta_i(k).
\tag{13}
$$

An optimal policy to minimize the one-step drift in (11) is to choose $\lambda_{\ell(k)}(k) = 1$ and $\lambda_j(k) = 0$ for all $j \ne \ell(k)$.

Remark 2: We will show in Section V how the notion of age-gain plays a central role also in the design of distributed age-based policies.

V. DECENTRALIZED AGE-BASED POLICIES

In this section, we propose a new class of decentralized policies designed to *prioritize* transmissions for the purpose of minimizing age of information. In each time slot k, transmitter i decides whether or not to send its packet depending on its local AoI, and in particular, based on $\delta_i(k)$ (defined in (12)).

To develop a deeper understanding of our proposed algorithm, let us focus on two regimes of operation assuming large M:

- The regime of infrequent arrivals, where $\theta \le \frac{1}{eM}$,
- The regime of frequent arrivals, where $\theta > \frac{1}{eM}$.

The choice of these two regimes is made based on the well-established performance of slotted ALOHA with respect to rate (throughput) [35, Chapter 4]. As explained earlier in Section I, we will first develop our framework for the slotted-ALOHA random access technology and then generalize to other random access technologies in Section V-E.

The basic idea of slotted ALOHA is as follows: At every time slot k, transmitters send their packets immediately upon arrival unless they are "backlogged" after a collision in which case they transmit with a backoff probability. In this section, we focus on Rivest's stabilized slotted ALOHA [35, Chapter 4]. In this algorithm, all arrivals are regarded as backlogged nodes that transmit with the backoff probability $p_b(k)$. Let $c(k) = 1$ denote the event that collision occurred at time k and $c(k) = 0$ denote the complementary event. The backoff probability is calculated through a pseudo-Bayesian algorithm based on an *estimate* of the number of backlogged nodes $n(k)$ (see [35, Chapter 4.2.3]):

$$p_b(k) = \min\left(1, \frac{1}{n(k)}\right)$$

$$n(k) = \begin{cases} n(k-1) + M\theta + (e-2)^{-1} & \text{if } c(k) = 1 \\ \max\left(M\theta, n(k-1) + M\theta - 1\right) & \text{if } c(k) = 0. \end{cases}$$
(14)

It is well known that this algorithm attains stability of queues for $\theta < \frac{1}{eM}$. In other words, transmitters can reliably send packets with a sum-rate up to $\frac{1}{e}$ in a decentralized manner [35, Chapter 4.2.3]. Asymptotically, when $M \to \infty$, the probability of delivering a packet in each time slot is $1/e$, the probability of collisions is $1 - 2/e$, and the probability of having an idle channel is $1/e$ (see Appendix E). Note that when $M\theta < \frac{1}{e}$, the expected total number of delivered packets in every time slot is $M\theta$.

We find the asymptotic NAAoI (in the limit of large M) in Theorem 1 below.

Theorem 1: Suppose $\theta < \frac{1}{eM}$ and define

$$\eta = \lim_{M \to \infty} M\theta.$$

Any stabilized slotted ALOHA scheme achieves

$$\lim_{M \to \infty} J^{SA}(M) = \frac{1}{\eta}.$$

Moreover, (stabilized) slotted ALOHA is order optimal in terms of NAAoI.

Proof: The proof is presented in Appendix F. The idea is to divide the sources into two groups in every time slot k: sources with $\delta_i(k) = 0$ and sources with $\delta_i(k) > 0$. We show that (i) the contribution of the first group of sources to NAAoI is equal to $\frac{1}{M\theta}$, and (ii) the second group constitutes only a vanishing fraction of the nodes and therefore, even though the sources in this group have larger $\delta_i(k)$'s, their total contribution vanishes as $M \to \infty$. □

A. Age-Based Thinning

When the arrival rate θ of each transmitter approaches $\frac{1}{eM}$, the NAAoI of slotted ALOHA approaches e (see Theorem 1). As θ increases beyond $\frac{1}{eM}$, the arrival rate gets larger than the maximum channel throughput ($= e^{-1}$), $n(k)$ overestimates the number of active transmitters and $p_b(k)$ underestimates the optimal probability of transmission, causing the throughput to decrease and the NAAoI to sharply increase.

Noting that the maximum channel rate/throughput is $\frac{1}{e}$ when (stabilized) slotted ALOHA algorithms are applied, a natural

question rises: What should the transmitters do in order to ensure a small age of information at the destination when $\theta \geq \frac{1}{eM}$? A naive solution to the above question would be to have each transmitter randomly drop packets and perform at the effective rate $\frac{1}{eM}$. But Theorem 1 shows that this only leads to NAAoI $\approx e$ which implies that we will not be able to benefit from the frequency of fresh packets to reduce age.

To benefit from the availability of fresh packets, we devise a decentralized age-based transmission policy in which transmitters prioritize packets that have larger age-gains. In particular, in each time slot k, transmitters find a common threshold $\text{T}(k)$ in order to distinguish and keep packets that offer high age-gains. The core idea is to still fully use the channel (depending on the available technology) but to carefully select, in a decentralized manner, what packets to send to minimize age. Recall that $\delta_i(k)$ denotes the age-gain of scheduling transmitter i. In our proposed algorithms, transmitters that have large age-gains become active and those with small age-gains stay inactive. More formally, Transmitter i is called *active* in time slot k if $\delta_i(k) \geq \text{T}(k)$. Only active transmitters participate in the transmission policy. Alternatively viewed, at time k, we propose to discard a fresh packet at transmitter i if $0 \leq \delta_i(k) < \text{T}(k)$ and to keep it otherwise. We refer to this process as thinning and this is done locally at the transmitters based on the AoI at the source/destination.

Note that no matter how the transmission policy is designed, since it is decentralized, it may happen that multiple transmitters try to access the channel at the same time, leading to collisions. For simplicity and clarity of ideas, we will restrict attention to slotted ALOHA techniques to resolve such collisions, and in particular the Rivest's stabilized slotted ALOHA[3] described in (14).

The main underlying challenge is in the design of $\text{T}(k)$. We propose two algorithms: an adaptive method of calculating $\text{T}(k)$ for each time slot based on the local collision feedback and a fixed threshold value T^* that is found in advance and remains fixed for all time slots k.

In the remainder of this section, we assume that M is large, and $\theta > \frac{1}{eM}$. The following definition comes in handy in presenting our results.

Definition 2: Consider transmitter i at time slot k. If $\delta_i(k) = m$, we say that transmitter i is an *m-order* node. Now let $\ell_m(k)$ be the expected fraction of m-order nodes in time slot k, i.e.,

$$\ell_m(k) = \mathbb{E}\left[\frac{1}{M} \sum_{i=1}^{M} 1_{\{\delta_i(k) = m\}}\right].$$
(15)

We define $\{\ell_m(k)\}_{m=0}^{\infty}$ as the average *node distribution (of the age-gain)* at time k.

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[3]In classical slotted ALOHA, "backlogged nodes" represent the nodes who have experienced collision and transmit with the backoff probability $p_b(k)$. In our version of Rivest's algorithm, since we have unit buffer sizes, we don't use the term "backlogged". We instead work with active nodes. In each time slot k, nodes decide based on their local age-gains whether they should be active. Active nodes transmit with probability $p_b(k)$, see (14) where $n(k-1)$ is the number of active nodes in time $k-1$.

B. Adaptive Threshold

Let $\mathtt{T}(k)$ denote the threshold for decision making in slot k. We propose to choose $\mathtt{T}(k)$ such that it imposes an effective arrival rate equal to $\frac{1}{eM}$ per transmitter. If the effective arrival rate per transmitter is less than $\frac{1}{eM}$, we are not utilizing the channel efficiently. If it is larger, then we are not prioritizing efficiently. This is because we would get a larger pool of packets than slotted ALOHA can support, leading to a reduced throughput and a larger age. More specifically, we design $\mathtt{T}(k)$ in three steps:

(i) Compute an estimate of the node distribution of the age-gain;
(ii) Find $\mathtt{T}(k)$ based on the estimated distribution;
(iii) Update the estimate of the node distribution based on the chosen $\mathtt{T}(k)$ and the collision feedback.

Note that $\{\ell_m(k)\}_{m=0}^{\infty}$ is unknown in decentralized systems. We hence find an estimate of it $\{\hat{\ell}_m(k)\}_{m=0}^{\infty}$ in every time slot. We summarize the process as follows

$$\{\hat{\ell}_m(k)\}_{m=0}^{\infty} = F(c(k), \{\hat{\ell}_m(k-1)\}_{m=0}^{\infty}) \qquad (16)$$

where $F(\cdot)$ is a function which will be determined later.

Suppose the estimated node distribution $\{\hat{\ell}_m(k-1)\}_m$ is known at (the end of) time slot $k-1$. We now describe how threshold $\mathtt{T}(k)$ is designed and how $\{\hat{\ell}_m(k)\}_m$ is updated. For clarity of ideas, let us view the time slot k in three stages: The first stage corresponds to the beginning of the time slot when new packets may arrive and replace the old packets. We denote the time just before the arrival of new packets by k^- and the time just after the arrival of packets by k^+. After the arrival of new packets, at time k^+, the source's AoI changes from $w_i(k^-)$ to $w_i(k^+)$ and the destination's AoI $h_i(k^+)$ remains the same as $h_i(k^-)$. So the age-gain values and their node distributions change. We denote the resulting node distribution in this stage by $\{\hat{\ell}_m(k^+)\}_m$. In the second stage, transmitters determine the threshold $\mathtt{T}(k)$ based on $\{\hat{\ell}_m(k^+)\}_m$. Transmissions happen according to the designed threshold $\mathtt{T}(k)$. In the third phase, at the end of time slot k when collision feedback is also available, the node distribution is once again estimated. We slightly abuse notation and denote the final estimate of the node distribution at the end of time slot k with $\{\hat{\ell}_m(k)\}_m$. The aforementioned three stages of calculating $\mathtt{T}(k)$ is described next.

Stage 1: Suppose the estimated node distribution $\{\hat{\ell}_m(k-1)\}_m$ is known at the beginning of slot k before the arrival of new packets. The expected fraction of m-order nodes that receive new packets is $\theta\hat{\ell}_m(k-1)$. The order of these nodes increase and this changes the expected node distribution to $\{\hat{\ell}_m(k^+)\}_m$ as a function of $\{\hat{\ell}_m(k-1)\}_m$. Let $a_m(k)$ denote the expected fraction of nodes that have just become m-order nodes at time k^+ for $m \geq 1$.

Lemma 1: The expected fraction of nodes that have just become m-order nodes at time k^+ is

$$a_m(k) = \theta^2 \sum_{j=0}^{m-1} \ell_j(k-1)(1-\theta)^{m-j-1} \qquad (17)$$

and the expected node distribution of age-gain at time k^+ is

$$\ell_m(k^+) = \begin{cases} (1-\theta)\ell_m(k-1) & m=0 \\ (1-\theta)\ell_m(k-1) + a_m(k) & m \geq 1. \end{cases} \qquad (18)$$

Proof: The proof is straightforward and delegated to Appendix G. □

We define $\hat{a}_m(k)$ as an estimate of $a_m(k)$, which can be obtained by (17) and (18) by replacing $\ell_m(k)$, $\ell_m(k^+)$ with $\hat{\ell}_m(k)$, $\hat{\ell}_m(k^+)$, respectively.

Stage 2: The threshold $\mathtt{T}(k)$ is determined based on $\{\hat{\ell}_m(k^+)\}_m$. We design $\mathtt{T}(k)$ such that the *effective arrival rate* of packets that have an age-gain above $\mathtt{T}(k)$ is close to $\frac{1}{e}$. In other words, we *thin* the arrival process using local age information. The critical point $\frac{1}{e}$ is the maximum sum arrival rate that ALOHA can support. So if the effective sum arrival rate falls below $\frac{1}{e}$, we do not use the full channel capacity[4] and if we operate above $\frac{1}{e}$, then we incur additional collisions and delay.

We define the effective arrival rate as the fraction of sources with new arrivals whose age-gain is larger than or equal to $\mathtt{T}(k)$. Recall that $\hat{a}_m(k)$ is the estimation of the expected fraction of nodes that have just become m-order nodes at time k^+ (coming from lower order nodes). So the total (estimated) fraction of nodes whose age-gain would, for the first time, pass the threshold $\mathtt{T}(k)$ is

$$\sum_{m \geq \mathtt{T}(k)} \hat{a}_m(k).$$

We propose to choose $\mathtt{T}(k)$ according to the following rule:

$$\mathtt{T}(k) = \max\left\{ t \mid \sum_{m \geq t} \hat{a}_m(k) \geq \frac{1}{eM} \right\}. \qquad (19)$$

Remark 3: We chose $\mathtt{T}(k)$ to be the maximum threshold value that does not bring effective sum arrival rate below $\frac{1}{e}$. This is due to the integer nature of age and hence k. One can also time share between $\mathtt{T}(k) - 1$ and $\mathtt{T}(k)$ to operate at an effective sum arrival rate (almost) equal to $\frac{1}{e}$. Thus, to simplify (14), we can replace $M\theta$ by the effective arrival rate $\frac{1}{e}$ in (14).

Remark 4: The threshold $\mathtt{T}(k)$ in (19) can also be applied to the regime $0 < \theta < \frac{1}{eM}$. In this regime, $\sum_{m=1}^{\infty} a_m(k) < \frac{1}{eM}$. Therefore, assuming that the estimates $\hat{a}_m(k)$ are accurate, the threshold is $\mathtt{T}(k) \leq 1$, reducing the proposed algorithm to the slotted ALOHA.

Stage 3: Once the threshold $\mathtt{T}(k)$ is determined, each transmitter verifies locally if its age-gain is above the specified threshold. If so, it transmits its packet with probability $p_b(k)$ defined in (14) mimicking slotted ALOHA. If collision happens or if all nodes abstain from transmitting, then AoI at the destination increases by 1 for all sources. If only one node transmits, its packet will be delivered successfully and the corresponding age at the destination drops to the source's AoI.

C. Estimating the Node Distribution

It remains to estimate $\hat{\ell}_m(k)$ at the end of time slot k, which will serve in computing $\mathtt{T}(k+1)$ in the next time slot.

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[4]Here, capacity refers to the maximum achievable sum rate under ALOHA.

We assume that at the end of time slot k, all transmitters are provided with collision feedback from the channel and we hence consider two cases separately: $c(k) = 0$ and $c(k) = 1$.

If collision has occurred, i.e., $c(k) = 1$, then the order of nodes will not change:

$$\hat{\ell}_m(k) = \hat{\ell}_m(k^+), \quad m \geq 0. \tag{20}$$

If there was no collision, i.e., $c(k) = 0$, then either a packet was delivered or no packet was delivered. Recall that we design $\mathtt{T}(k)$ to impose (in the limit of large M) an effective sum arrival rate almost equal to $\frac{1}{e}$. Following Lemma 3 in Appendix E, the two events of idle and successful delivery are almost equiprobable for large M:

$$\lim_{k \to \infty} \Pr \Big(\sum_{i=1}^{M} d_i(k) = 1, c(k) = 0 \Big) \approx \frac{1}{e}$$

$$\lim_{k \to \infty} \Pr \Big(\sum_{i=1}^{M} d_i(k) = 0, c(k) = 0 \Big) \approx \frac{1}{e}.$$

Thus, condition on $c(k) = 0$, a packet is delivered with probability $1/2$, i.e., the expected number of delivered packet is $1/2$ and by the inherent symmetry of the system, each active node has the same chance to deliver a new packet. For any $m \geq \mathtt{T}(k)$, a packet is delivered by m-order nodes with probability

$$r_m(k) = \frac{\ell_m(k^+)}{\sum_{t \geq \mathtt{T}(k)} \ell_t(k^+)}. \tag{21}$$

The expected number of m-order nodes is $M\ell_m(k^+)$ and the expected number of delivered packets by m-order nodes (condition on $c(k) = 0$) is $\frac{r_m(k)}{2}$. Note that m-order nodes can not deliver more than $M\ell_m(k^+)$ packets since the total number of m-order nodes is $M\ell_m(k^+)$ and the buffer size is 1, then

$$\frac{r_m(k)}{2} < M\ell_m(k^+). \tag{22}$$

In order to estimate the expected fraction of m-order nodes that have a successful delivery, we simply plug in $\hat{\ell}_m(k^+)$ as an estimate for $\ell_m(k^+)$. Since (22) does not necessarily hold anymore using the estimates, we estimate the expected fraction of m-order nodes with a successful delivery as follows:

$$\frac{1}{M} \min \Big(\frac{r_m(k)}{2}, M\hat{\ell}_m(k^+) \Big),$$

where $r_m(k)$ is computed by (21) and replacing $\ell_m(k)$ with $\hat{\ell}_m(k)$. Consequently, the update rule of the node distribution of age, $\{\hat{\ell}_m(k)\}_m$, is given as follows:

$$\hat{\ell}_0(k) = \hat{\ell}_0(k^+) + \sum_{m=\mathtt{T}(k)}^{\infty} \min \Big(\frac{r_m(k)}{2M}, \hat{\ell}_m(k^+) \Big)$$

$$\hat{\ell}_m(k) = \hat{\ell}_m(k^+), \qquad 1 \leq m \leq \mathtt{T}(k) - 1$$

$$\hat{\ell}_m(k) = \Big(\hat{\ell}_m(k^+) - \frac{r_m(k)}{2M} \Big)^+, \qquad m \geq \mathtt{T}(k). \tag{23}$$

Collecting Stages 1 - 3, from (18), (16) can be re-written as

$$\hat{\ell}_0(k) = (1 - \theta)\hat{\ell}_0(k - 1) + 1_{\{c(k)=0\}}$$
$$\times \sum_{m=\mathtt{T}(k)}^{\infty} \min \Big(\frac{r_m(k)}{2M}, (1 - \theta)\hat{\ell}_m(k - 1) + \hat{a}_m(k) \Big)$$

$$\hat{\ell}_m(k) = (1 - \theta)\hat{\ell}_m(k - 1) + \hat{a}_m(k), \quad 1 \leq m \leq \mathtt{T}(k) - 1$$

$$\hat{\ell}_m(k) = \Big((1 - \theta)\hat{\ell}_m(k - 1) + \hat{a}_m(k) - 1_{\{c(k)=0\}} \frac{r_m(k)}{2M} \Big)^+,$$
$$m \geq \mathtt{T}(k). \tag{24}$$

where $a_m(k)$ and $r_m(k)$ are defined in (17) and (21), respectively. Finally, in this case, the probability of transmission is calculated by (14), where $M\theta$ is replaced by the effective arrival rate $\frac{1}{e}$.

Algorithm 1 describes the proposed distributed age-based transmission policy. We numerically evaluate its age performance in Section VI and analyze a stationary version of it when the threshold is fixed in Section V-D. Comparing with the slotted ALOHA in (14), Algorithm 1 significantly reduces the NAAoI when the sum arrival rate is beyond $\frac{1}{e}$ (see Fig. 3a and Fig. 4a). It achieves this by carefully selecting and delivering packets with a large age-gain. The NAAoI under Algorithm 1 decreases sharply when the arrival rate θ approaches 1 (see Figure 2a). In particular, the NAAoI it achieves at $\theta = 1$ is almost 1. Contrasting that with the lower bound in Proposition 1, one comes to the conclusion that the throughput achieved by Algorithm 1 is larger than that of a standard slotted ALOHA. This is because of the implicit coordination that is facilitated by estimating and utilizing the age gain distributions for decision making.

Algorithm 1 Adaptive Age-Based Thinning (AAT)

Set a large integer N and the time horizon K.
Set initial points: $h_i(0) = 1$, $w_i(0) = 0$ for $i = 1, 2, \cdots, M$; $c(0) = 0$; $\mathtt{T}(0) = 1$; $p_b(0) = 1$; $n(0) = 0$; $k = 1$.
repeat
  Step 1: Calculate $\{\hat{\ell}_m(k^+)\}_{m=1}^N$ by (18).
  Step 2: Calculate $\mathtt{T}(k)$ by (19).
  Step 3: For transmitter i, $i = 1, \ldots, M$: compute $\delta_i(k^+) = h_i(k^+) - w_i(k^+)$; if $\delta_i(k^+) < \mathtt{T}(k)$, then it does not transmit packets; if $\delta_i(k^+) \geq \mathtt{T}(k)$, then it transmits a packet with probability $p_b(k)$[5].
  Step 4: If $c(k) = 0$, calculate $\{\hat{\ell}_m(k)\}_{m=1}^N$ by (20), and if $c(k) = 1$, calculate $\{\hat{\ell}_m(k)\}_{m=1}^N$ by (23).
  Calculate $p_b(k + 1)$ by (14) in which $M\theta$ is replaced by $\min(M\theta, e^{-1})$.
until $k = K$
Calculate J_K^{AAT} by (3).

Remark 5: From (14), to estimate the number active nodes in each time slot, the number of nodes in the network is needed. We set M to be a pre-determined parameter, which is known to all nodes.

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[5]A packet with a large age-gain must have a packet ready to transmit.

Remark 6: The estimates $\{\hat{\ell}_m(k)\}_m$ and $\{\hat{a}_m(k)\}_m$ in Algorithm 1 are not exactly accurate and this is due to the integer nature of the threshold. Assume that $\{\hat{\ell}_m(k_0)\}_m$ and $\{\hat{a}_m(k_0)\}_m$ are exactly accurate in time slot k_0. We may have $\sum_{m \geq \mathtt{T}(k_0)+1} \hat{a}_m(k_0) < \frac{1}{eM}$ but $\sum_{m \geq \mathtt{T}(k_0)} \hat{a}_m(k_0) > \frac{1}{eM}$ in which case the effective arrival rate $\left(= \sum_{m \geq \mathtt{T}(k_0)} \hat{a}_m(k_0)\right)$ would be larger than e^{-1}. But the steps in (23) are derived by assuming an effective arrival rate $\frac{1}{e}$ and this leads to inaccuracies in our estimates $\{\hat{\ell}_m(k)\}_m$ and $\{\hat{a}_m(k)\}_m$ as computed in Algorithm 1.

Remark 7: We updated $\{\hat{\ell}_m(k)\}_m$ as a function of $\{\hat{\ell}_m(k-1)\}_m$ and the collision feedback $c(k)$, hence the name adaptive. $\mathtt{T}(k)$ and $\{\hat{\ell}_m(k)\}_m$ are known at all sources and every source finds the same $\mathtt{T}(k)$. If we update $\{\ell_m(k)\}_m$ (not $\{\hat{\ell}_m(k)\}_m$) by the conditional expectation of $\{\ell_m(k)\}_m$, condition on $\{\ell_m(k^+)\}_m$ but not on $c(k)$, we will find a fixed limiting threshold $\mathtt{T}^*$ discussed next.

D. Fixed Threshold

A simple variant of the age-based thinning method is found when the threshold $\mathtt{T}(k) = \mathtt{T}^*$ is fixed throughout the transmission phase. In particular, we design $\mathtt{T}^*$ ahead of time based on the node distribution in the stationary regime. By doing so, we cannot benefit from the collision feedback to adaptively choose $\mathtt{T}(k)$. However, this framework is preferable for deriving analytical results.

We use the framework and derivation we developed for adaptive thinning in order to find a fixed "optimal" $\mathtt{T}^*$ that imposes an effective arrival rate approximately[6] equal to $1/e$. Note that a larger arrival rate implies further random thinning of the packets to meet the fundamental rate $1/e$ (as opposed to the selective nature of thinning by imposing an age thresholding) and a smaller arrival rate corresponds to inefficient utilization of the channel.

The major difference between an adaptive threshold and a fixed threshold is in the update rules (20)-(23) because $c(k)$ is not known when $\mathtt{T}^*$ is designed. In particular, the update rule (20)-(23) is replaced by an average rule that weighs $c(k) = 1$ with probability $1 - \frac{2}{e}$ and $c(k) = 0$ with probability $\frac{2}{e}$ (following Lemma 3).

By the stationarity of the scheme, the limit of $\{\ell_m(k)\}_{m=0}^{\infty}$ and $\{\ell_m(k^+)\}_{m=0}^{\infty}$ exist as $k \to \infty$. Denote the two limits by $\{\ell_m^*\}_{m=0}^{\infty}$ and $\{\ell_m^{+*}\}_{m=0}^{\infty}$, respectively. Similar with (18), the update rule of Stage 1 implies

$$\ell_0^{+*} = (1-\theta)\ell_0^*$$
$$\ell_m^{+*} = (1-\theta)\ell_m^* + a_m^* \qquad m \geq 1 \qquad (25)$$

where

$$a_m^* = \theta^2 \sum_{j=0}^{m-1} \ell_j^* (1-\theta)^{m-j-1} \qquad m \geq 1. \qquad (26)$$

Since we let $\mathtt{T}(k) = \mathtt{T}(k-1) = \mathtt{T}^*$, the threshold proposed in Stage 2 is

$$\mathtt{T}^* = \max\left\{t \Big| \sum_{m \geq t} a_m^* \geq \frac{1}{eM}\right\}. \qquad (27)$$

[6]This approximation is due to the integer nature of the age threshold.

Next, consider Stage 3. In contrast to Section V-C, we do not utilize collision feedback in finding $\mathtt{T}(k)$. So estimating the fraction of m-order nodes at the end of time slot k will account for $c(k) = 1$ with probability $1 - \frac{2}{e}$ and $c(k) = 0$ with probability $\frac{2}{e}$ (see Lemma 3). We hence obtain

$$\ell_0^* = \ell_0^{+*} + \frac{1}{eM}$$
$$\ell_m^* = \ell_m^{+*}, \qquad\qquad 1 \leq m \leq \mathtt{T}^* - 1$$
$$\ell_m^* = \ell_m^{+*} - \frac{r_m^*}{eM}, \qquad m \geq \mathtt{T}^* \qquad (28)$$

where

$$r_m^* = \ell_m^{+*} / \sum_{i=\mathtt{T}}^{\infty} \ell_i^{+*}.$$

Putting together (25) - (28), we obtain

$$\ell_0^* = (1-\theta)\ell_0^* + \frac{1}{eM}$$
$$\ell_m^* = (1-\theta)\ell_m^* + a_m^*, \quad 1 \leq m \leq \mathtt{T}^* - 1$$
$$\ell_m^* = (1-\theta)\ell_m^* + a_m^* - \frac{r_m^*}{eM}, \quad m \geq \mathtt{T}^* \qquad (29)$$

and conclude the following lemma (see Appendix H for the proof).

Lemma 2: As $k \to \infty$, the stationary distributions $\{\ell_m^*\}_m$, $\{\ell_m^{+*}\}_m$ and $\{a_m^*\}_m$ satisfy the following properties:

$$\ell_m^* = \begin{cases} \frac{1}{eM\theta} & m = 0 \\ \frac{1}{eM} & 1 \leq m \leq \mathtt{T}^* - 1 \end{cases} \qquad (30)$$

$$\ell_m^{+*} = \frac{1}{eM} \qquad 1 \leq m \leq \mathtt{T}^* - 1 \qquad (31)$$

$$a_m^* = \frac{\theta}{eM} \qquad 1 \leq m \leq \mathtt{T}^*. \qquad (32)$$

The closed form expression of the fixed threshold $\mathtt{T}^*$ is given below (see Appendix I for the proof) and Algorithm 2 describes our stationary ge-based transmission policy.

Theorem 2: The fixed threshold $\mathtt{T}^*$ in (27) has the following closed form expression:

$$\mathtt{T}^* = \lfloor eM - \frac{1}{\theta} + 1 \rfloor.$$

Remark 8: The threshold in Theorem 2 can be applied to the regime $0 < \theta < \frac{1}{eM}$ as well. In particular, in this regime, the threshold is $\mathtt{T}^* \leq 0$ and the proposed algorithm reduces to the slotted ALOHA.

We finally prove asymptotically (as $M \to \infty$) that the Stationary Age-based Thinning (SAT) policy described in Algorithm 2 significantly reduces age when $1/\theta = o(M)$. Recall that at $\theta = \frac{1}{eM}$, we have $\lim_{M \to \infty} J^{SA}(M) = e$. For larger arrival rates θ where $1/\theta = o(M)$, we prove that Algorithm 2 sharply reduces AoI from e to $\frac{e}{2}$.

Theorem 3: For any $\theta = \frac{1}{o(M)}$,

$$\lim_{M \to \infty} J^{SAT}(M) = \frac{e}{2}.$$

Remark 9: The minimum NAAoI attained by a stabilized slotted ALOHA is (asymptotically) e and it is achieved at $\theta = \frac{1}{eM}$ (See Theorem 1 and Fig. 4a). Theorem 3 shows that our proposed SAT policy attains the NAAoI $\frac{e}{2}$ (asymptotically) for $\theta = \frac{1}{o(M)}$. This provides a multiplicative factor of

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Algorithm 2 Stationary Age-Based Thinning (SAT)

Set the time horizon K.

Set initial points: $h_i(0) = 1$, $w_i(0) = 0$ for $i = 1, 2, \cdots, M$; $c(0) = 0$; $p_b(0) = 1$; $n(0) = 0$; $k = 1$.

Calculate $\mathtt{T}^* = \lfloor eM - \frac{1}{\theta} + 1 \rfloor$.

repeat

  Step 1: For transmitter i, compute $\delta_i(k) = h_i(k) - w_i(k)$, if $\delta_i(k) < \mathtt{T}^*$, then it does not transmit packets; if $\delta_i(k) \geq \mathtt{T}^*$, then it transmits a packet with probability $p_b(k)$.

  Step 2: Calculate $p_b(k+1)$ by (14) in which $M\theta$ is replaced by $\min(M\theta, e^{-1})$.

until $k = K$

Calculate J_K^{SAT} by (3).

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2 compared to the minimum NAAoI under slotted ALOHA. Moreover, simulation results show that the AAT policy outperforms the SAT policy for $\theta = \frac{1}{o(M)}$ (see Fig. 4a).

Proof: The proof is given in Appendix J. Here, we provide the road-map of the proof. In every time slot k, the sources can be divided into two groups: 1) sources with $\delta_i(k) < \mathtt{T}^*$; 2) sources with $\delta_i(k) \geq \mathtt{T}^*$. The first group of sources have the main contribution to $J^{SAT}(M)$ (which is equal to $\frac{e}{2}$) when $M \to \infty$. The contribution of the second group of sources to $J^{SAT}(M)$ vanishes when $M \to \infty$. $\square$

E. Extensions to Other Random Access Technologies

So far, we restricted attention to slotted ALOHA as the main random access technology. However, in the past decade, novel technologies such as Carrier Sensing Multiple Access (CSMA) technologies have emerged and led to significant improvements in terms of throughput. It is interesting to know how they perform with regard to age, especially since they are known to have large delays [30], [31], [37]–[42]. In this regard, [30], [31] have proposed an efficient sleep-wake mechanism for wireless networks that attains the optimal trade-off between minimizing the AoI and energy consumption. In [40], a network with M sources (links) under CSMA scheme was considered and the closed form of average age of information was derived as a function of the back-off time and generation rate. In [41], the notion of broadcast age of information was investigated in wireless networks with CSMA/CA technologies.

In this section, we outline how the age-based thinning method described in Section V-D (with a fixed threshold) can be applied to other random access technologies. For this purpose, we consider any transmission policy π that does not employ coding across packets. All existing collision avoidance and resolution techniques such as ALOHA and CSMA [43]–[46] fall into this class. Now develop a variant of the transmission policy π in which only the most recent packets of each transmitter are preserved and all older packets are discarded. Denote this policy by $\pi^{(1)}$. Define $C^{\pi^{(1)}}(M)$ as the maximum sum throughput when applying the transmission policy $\pi^{(1)}$ in a system with M sources, and denote the limit, when $M \to \infty$, by $C^{\pi^{(1)}}$. Consider the age-based thinning

process in two steps: (i) the threshold $\mathtt{T}^*$ is calculated, (ii) all nodes with age-gains larger than or equal to $\mathtt{T}^*$ become active and transmit using the prescribed random access technology.[7]

Consider M to be large, and suppose the expected number of delivered packets per time slot is around $\min(M\theta, C^{\pi^{(1)}})$. Therefore, (25) remains the same and (27) takes the following form:

$$\mathtt{T}^* = \max \left\{ t \Big| \sum_{m \geq t} a_m^* \geq \frac{C^{\pi^{(1)}}}{M} \right\}. \tag{33}$$

Following a similar argument as in Section V-D, the equations in (28) can be written more generally as follows:

$$
\begin{aligned}
\ell_0^* &= \ell_0^{+*} + \min(\theta, \frac{C^{\pi^{(1)}}}{M}) \\
\ell_m^* &= \ell_m^{+*} & 1 \leq m \leq \mathtt{T}^* - 1 \\
\ell_m^* &= \ell_m^{+*} - r_m^* \min(\theta, \frac{C^{\pi^{(1)}}}{M}) & m \geq \mathtt{T}^*
\end{aligned}
\tag{34}
$$

where

$$r_m^* = \ell_m^{+*} / \sum_{i=\mathtt{T}^*}^{\infty} \ell_i^{+*}.$$

Combining (25), (33), (34), we thus find

$$\ell_0^* = \begin{cases} \min(1, \frac{C^{\pi^{(1)}}}{M\theta}) & m = 0 \\ \min(\theta, \frac{C^{\pi^{(1)}}}{M}) & 1 \leq m \leq \mathtt{T}^* - 1 \end{cases} \tag{35}$$

$$\ell_m^* = \min(\theta, \frac{C^{\pi^{(1)}}}{M}) \qquad 1 \leq m \leq \mathtt{T}^* - 1 \tag{36}$$

$$a_m^* = \min(\theta^2, \frac{\theta C^{\pi^{(1)}}}{M}) \qquad 1 \leq m \leq \mathtt{T}^*. \tag{37}$$

Moreover, the threshold T^* takes a simple closed-form expression as stated below (and proved in Appendix K).

Theorem 4: The fixed threshold $\mathtt{T}^*$ in (33) has the following closed form expression:

$$\mathtt{T}^* = \left\lfloor \frac{M}{C^{\pi^{(1)}}} - \frac{1}{\theta} + 1 \right\rfloor.$$

Using this result, Algorithm 3 proposes a decentralized age-based thinning method that can be applied to any given stationary random access technology.

We prove an analogue to Theorem 3, showing that the Generalized Stationary Age-based Thinning policy (GSAT) proposed in Algorithm 3 reduces age to $\frac{1}{2C^{\pi^{(1)}}}$ as θ increases.

Theorem 5: For any $\theta = \frac{1}{o(M)}$,

$$\lim_{M \to \infty} J^{GSAT}(M) = \frac{1}{2C^{\pi^{(1)}}}.$$

Proof: The proof of Theorem 5 is given in Appendix L. $\square$

Remark 10: The results in this section are stronger than [29] in three aspects: (i) we gave a simple and explicit expression for the threshold $\mathtt{T}^*$, while the threshold has to be computed numerically in [29]; (ii) we found the asymptotical NAAoI ($\lim_{M \to \infty} \mathbb{E}[J^{GSAT}(M)]$) analytically; (iii) the

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[7]Here, "prescribed random access technology" refers to the specific transmission scheme which is applied to the random access channel.
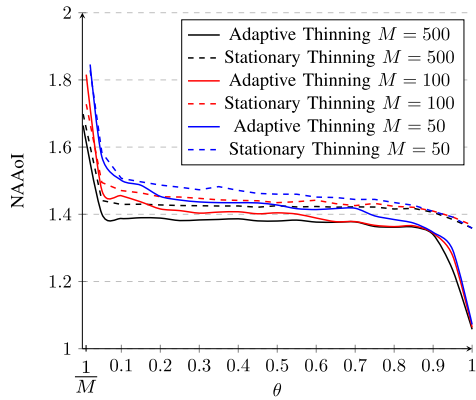
(a) NAAoI for stationary and adaptive age-based policies.
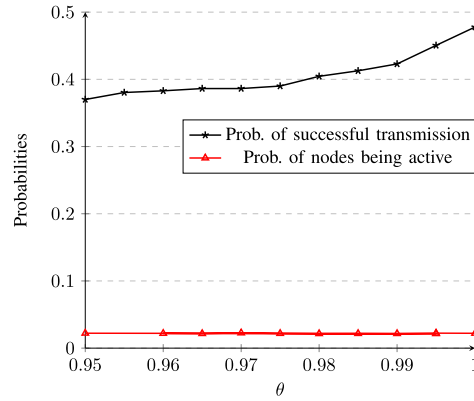
(b) Probabilities of successful transmission and being active in adaptive thinning policies.

Fig. 2. NAAoI and success transmission probabilities.

Algorithm 3 Generalized Stationary Age-Based Thinning (GSAT)

Set the time horizon K.
Set initial points: $h_i(0) = 1$, $w_i(0) = 0$ for $i = 1, 2, \cdots, M$;
$c(0) = 0$; $p_b(0) = 1$; $n(0) = 0$; $k = 1$.
Calculate the threshold $\mathtt{T}(C^{\pi^{(1)}}) = \lfloor \frac{M}{C^{\pi^{(1)}}} - \frac{1}{\theta} + 1 \rfloor$.
repeat
  For the source node i, compute $\delta_i(k) = h_i(k) - w_i(k)$.
  If $\delta_i(k) < \mathtt{T}(C^{\pi^{(1)}})$ remain silent; If $\delta_i(k) \geq \mathtt{T}(C^{\pi^{(1)}})$, transmits according to the random access technology $\pi^{(1)}$.
until $k = K$
Calculate J_K^{GSAT} by (3).

---

threshold in this section can be applied not only to CSMA, but also to any other transmission policy.

Remark 11: The framework we have built, particularly Algorithm 3 and Theorem 5, can be directly applied to other settings and multi-access technologies such as MAC with common information in [47] and a queue-length-based MAC in [52]. These technologies can achieve sum capacity 1, like CSMA, and their corresponding normalized age tends to $\frac{1}{2}$ as M gets large.

VI. NUMERICAL RESULTS

In this section, we verify our findings through simulations. Figure 2a shows the normalized age under adaptive and stationary age-based transmission policies for $M = 50, 100, 500$. For stationary age-based policies, the normalized age converges to $\frac{e}{2}$ when M is large, validating our findings in Theorem 3.

The performance of the adaptive policy is better than that of the stationary age-based policy for $\theta > \frac{1}{M}$ and the efficacy (the gap between the two curves) increases with θ. Since the maximum sum throughput of slotted ALOHA is $\frac{1}{e}$, one may ask if this contradicts the lower bound of Proposition 2. To answer this question, we remark that the adaptive age-based transmission policy is *not* a slotted ALOHA scheme and

therefore the maximum throughput of slotted ALOHA would not apply. As a matter of fact, Fig. 2b shows that the throughput of the scheme increases beyond $\frac{1}{e}$ with θ, supporting Proposition 1. This is because the AAT policy implicitly facilitates coordination among the transmitters as they utilize the (estimated) age-age distribution for decision making. The throughput at $\theta = 1$, as seen in Fig. 2b, is close to .48 which is consistent with the known lower bound 0.4878 and upper bound 0.568 on the (information theoretic) channel capacity of random access channels with feedback [36], [53]. It is interesting that the AAT policy can both increase the throughput and decrease the AoI simultaneously when θ approaches 1.

One can also observe that the adaptive policy performs worse than the stationary policy for $\frac{1}{eM} \leq \theta \leq \frac{1}{M}$. The SAT policy is designed as the stationary version of the AAT policy, thus the SAT policy should not in principle outperform the AAT policy (assuming that our approximations of the estimates of the age-gain distribution are accurate). However, it is worthwhile to discuss this counter-intuitive phenomenon and we expand on the underlying reason: We consistently underestimate the threshold $\mathtt{T}(k)$ due to the integer nature of it, especially when θ is small. For example, consider $\frac{1}{eM-1} \leq \theta < \frac{1}{eM-2}$. Note that $\sum_{m \geq 1} a_m(k) = \theta$, which is close to $\frac{1}{eM}$. From (19), in some slots, the event $\{\sum_{m \geq 2} \hat{a}_m(k) < \frac{1}{eM}\}$ may occur (even if the estimate $\{\hat{a}_m(k)\}_m$ is exactly accurate), so the threshold $\mathtt{T}(k)$ under the AAT policy would be 1 because $\mathtt{T}(k)$ is always an integer. In these time slots, the AAT policy is reduced to the slotted ALOHA. Therefore, the fraction of active nodes becomes large, the throughput decreases, and the age increases. On the other hand, in the regime when $\frac{1}{eM} \leq \theta \leq \frac{1}{M}$, the threshold $\mathtt{T}^* = 2$ under the SAT policy in every time slot (see Theorem 2). Subsequently, the estimate of the age-gain distribution $\{\hat{\ell}_m(k)\}_m$ is imprecise (see Remark 6). Moreover, the imprecise estimates in (24) may aggravate the underestimation of the threshold, and the closed-loop worsens the performance of the AAT policy. The effect is more pronounced in the regime $\frac{1}{eM} \leq \theta \leq \frac{1}{M}$ where old packets are less frequently replaced with new packets.
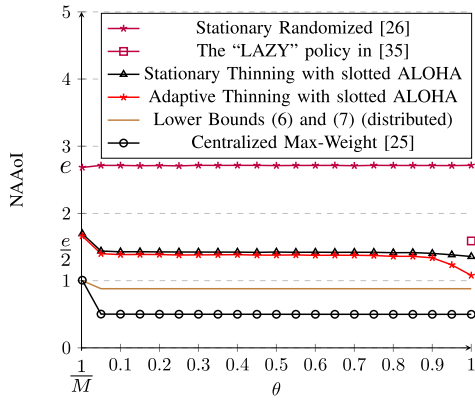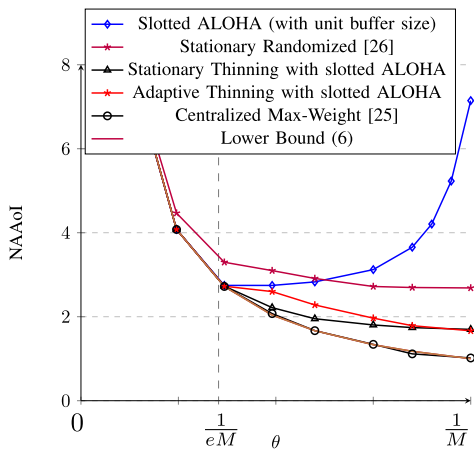
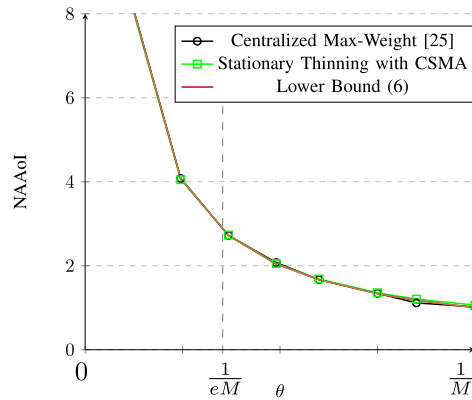(a) NAAoI of slotted ALOHA and its comparisons
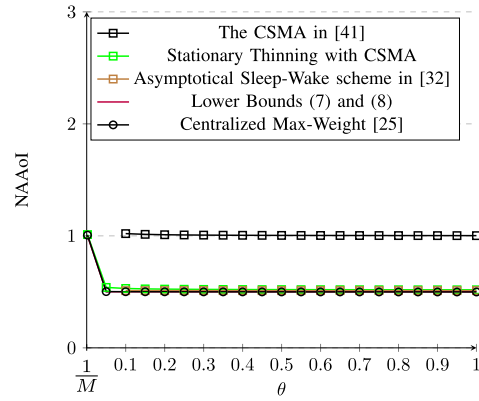
(b) NAAoI of CSMA and its comparisons

Fig. 3. NAAoI when $M = 500$ v.s $\theta \in (\frac{1}{M}, 1]$.

(a) NAAoI of slotted ALOHA and its comparison

(b) NAAoI of Stationary thinning with CSMA

Fig. 4. NAAoI when $M = 500$ v.s $\theta \in (0, \frac{1}{M}]$.

Finally the age-performance of our proposed distributed age-based policies are compared with the lower bounds of Section III, state-of-the-art distributed schemes such as [25], as well as centralized Max-Weight policies such as [24]. For clarity of exposition, we consider two regimes of θ: $\theta \in (\frac{1}{M}, 1]$ (see Fig. 3), and $\theta \in (0, \frac{1}{M}]$ (see Fig. 4). Fig. 4, in particular, shows that when $\theta \leq \frac{1}{eM}$, the normalized age of slotted ALOHA coincides with centralized Max-Weight policies and the lower bound of Proposition 2. When θ increases beyond $\frac{1}{eM}$, our proposed age-based thinning methods provide significant gains compared to randomized stationary and slotted ALOHA schemes. When $\theta = \frac{1}{o(M)}$, the NAAoI of slotted ALOHA explodes, and we omit the curve of slotted ALOHA in Fig. 3a. Finally, we numerically observe that the normalized age of the centralized Max-Weight policy is approximately attained by stationary age-based thinning in perfect CSMA[8] (see the green square curve in Fig. 3b and Fig. 4b), where the length of one contention slot is set to be 1/100 [29], [54].

Next, we compare our proposed algorithms with policies in related works, such as a lazy version of slotted aloha in [34], and variants of CSMA in [30], [31], [40]. Different from [34], we considered a random access channel with re-transmission attempts for packets and used a slotted ALOHA with time-variant transmission probability, while in [34], a channel without re-transmission and a slotted ALOHA with a time-invariant transmission probability is investigated. More importantly, we proposed a policy where the best threshold is found in every time slot, while a predetermined threshold is given in [34]. Furthermore, we showed the performance analysis for arbitrary M sources under arbitrary generation/arrival rate (in $[0, 1]$), while [34] only provided the closed form of average AoI for the case when $M = 2$ and $\theta = 1$. Compared to the performance of the policy in [34], our proposed AAT and SAT policies outperform the lazy version of slotted aloha (see Fig. 3a, the purple square curve). To apply the policies in [40] on our model, we consider the generation/arrival rate is relatively large ($\theta \geq 0.1$) because under [40, Assumption 1], a transmitter sends a "fake" update if its buffer is empty. From Figure 3b, it is easy to see that the stationary thinning with CSMA outperforms the policy

[8]We consider perfect CSMA in simulations. In other words, no errors occur in carrier sensing.

in [40] (Fig 3b, square black curve). References [30], [31] are concurrent works on optimizing peak AoI over random access channels with per-source battery lifetime constraints. Translating the introduced energy constraints to arrival rate θ, one can apply the sleep-waking schemes of [30], [31] to our problem when CSMA capabilities are available. Using the symmetry of our model along with [31, Eqns. (13), (16)], when M is sufficiently large, the fraction of time in which every source is in transmission mode is around $r/(Mr+1)$ where r is the sleep period parameter as proposed in [30], [31]. Using [31, Eqns. (13), (16)], one can argue that we need to consider the so called energy-adequate regime introduced in [30], [31] which translates to relatively large θ, i.e., $\theta \geq 0.1$. The performance of the policy in [30], [31] (Fig 3b, red square point) is similar to that of the stationary thinning with CSMA, which is consistent with the optimality results presented in [30], [31].

It is worthwhile to mention that the proposed algorithms not only utilize fully channel capacity, but minimize NAAoI. If we only consider policies with maximum throughput (e.g. standard slotted ALOHA and its variants), the NAAoI explodes up with time for arrival rates above $\frac{1}{e}$. This is also observed in works such as [25] that adapt slotted ALOHA without packet management for age minimization.

VII. CONCLUSION AND FUTURE RESEARCH

In this work, we investigated the AoI performance of a *decentralized* system consisting M source nodes communicating with a common receiver. We first derived a general lower bound on AoI. Then, we derived the analytical (normalized) age performance of (stabilized) slotted ALOHA in the limit of $M \to \infty$. As the sum arrival rate increases beyond $\frac{1}{e}$, slotted ALOHA becomes unstable. We show that by prioritizing transmissions that offer significant reduction in AoI, we can increase the arrival rate and simultaneously decrease AoI. In particular, we proposed two age-based thinning policies: (i) Adaptive Age-based Thinning (AAT) and (ii) Stationary Age-based Thinning (SAT) and analyzed the age performance in the limit of $M \to \infty$. Finally, we demonstrated how our proposed thinning mechanism (SAT) is useful for other random access technologies. Numerical results showed that the proposed age-based thinning mechanisms make a significantly contribution to the performance of age even for moderate values of M. Our framework can not be extended to generalized settings (such as [48]–[51]) blindly. Appropriate adaptations related to different settings are necessary. For example, after applying Algorithm 3 to transmission schemes in [49], [50], they are reduced to stationary randomized policies. This is because we do not fully utilize the additional knowledge of the queue length that is provided in these settings. In the setting of [48], we should further assume that the set of nodes that transmit packets are known, and a continuous-time version of the framework is needed in the setting of [51].

Future research includes generalization to accommodate 1) dynamic channels, i.e., the number of nodes M, or the arrival rates θ are time-variant 2) asymmetric channels, i.e., the arrival rates θ_i is different. In the first case, the method we proposed above can be applied directly. Suppose that the expressions of the number of nodes, $M(k)$, and the arrival rates, $\theta(k)$, are known. We cane replace M and θ by $M(k)$ and $\theta(k)$, respectively, in every time slot. Subsequently, the fixed threshold hold T^* is also a time-variant variable, $T^*(k)$. In the second case, the method we proposed above can not be applied directly. This is because we use the profile of all sources as an estimate on any individual source. A more general estimation method should be proposed in the second case. In addition, note that slotted ALOHA algorithms are order optimal when M is sufficiently large and generation/arrival rate is small ($\theta \leq \frac{1}{eM}$). An interesting extension is to consider a smarter decentralized age-based algorithm that can achieve a constant additive age gap from the optimum average age when θ is small.

APPENDIX A
SUFFICIENCY OF UNIT BUFFER SIZE

Consider two types of policies: policies with buffer size 1, denoted by π_1, and policies with larger buffer sizes, denoted by π_2. To differentiate the two policies and their corresponding queues, we label the packets inside the queues by *new* and *old*. A new packet in a queue refers to the latest arrival. A packet in a queue is considered old if there is a newer packet in the same queue or if the packet (or a fresher packet) from that source is already delivered at the receiver. In the following, we refer to the freshest old packet as the old packet. At a given time slot, denote the new packet and the old packet of source i by $p_{new}^{(i)}$ and $p_{old}^{(i)}$, respectively. Denote the arrival times of the new and old packets as $t_i^{(n)}$ and $t_i^{(o)}$. It is clear that $t_i^{(n)} > t_i^{(o)}$. We will show that no matter what policy π_2 does, there is always a policy of type π_1 whose resulting age is at least as low as π_2 with respect to every source node.

At time slot t', suppose policy π_2 chooses certain action, then we design policy π_1 to follow the same action with the new packet. In this time slot, under π_2 a subset of sources transmit packets. Denote the index of these sources by $\mathcal{I}$. For the sources which do not transmit packets, the AoI under both policies will increase by 1. For the sources in $\mathcal{I}$, we have the following two cases:

Case 1: Suppose collision happens in time slot t'. Then, no packet is delivered, and the AoI of these sources under both policies will increase by 1.

Case 2: If a packet is delivered, which implies the cardinal of $\mathcal{I}$, $|\mathcal{I}| = 1$. Denote the index of this source by i. Then at the next time slot, the AoI under π_1 drops to $h_i^{\pi_1}(t'+1) = t' - t_i^{(n)} + 1$, and the AoI under π_2 drops to $h_i^{\pi_2}(t'+1) = t' - t_i^{(o)} + 1 > h_i^{\pi_1}(t'+1)$. This means that from t' onward $h_i^{\pi_2}(t)$ will be point-wise larger or equal to $h_i^{\pi_1}(t)$, $t > t'$.

APPENDIX B
PROOF OF PROPOSITION 1

Consider any transmission policy and a large time-horizon K. Let L_i be the number of remaining time slots after the last packet delivery in source i. The NAAoI defined in (3) can be re-written as follows:

$$J_K^\pi = \frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \sum_{k=1}^{K} h_i(k)$$

$$= \frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \left(\sum_{m=1}^{N_i(K)} \Gamma_i(m) + \frac{1}{2} L_i^2 \right.$$
$$\left. + D_i(N_i(K)) L_i - \frac{1}{2} L_i \right), \tag{38}$$

where $\Gamma_i(m)$ was expressed in (4). Since $D_i(m) \geq 1$ for all $1 \leq m \leq N_i(K)$, we can lower bound $\Gamma_i(m)$ by substituting $D_i(m-1) = 1$ in (4). Using similar steps as [17, Eqns. (9) - (14)], we find

$$J^\pi(M) \geq \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{2M^2} \sum_{i=1}^{M} \frac{K}{N_i(K)} + \frac{1}{2M} \right]. \tag{39}$$

Recall that $N_i(K)$ is the total number of packets delivered by source i. In the limit of $K \to \infty$, $\frac{N_i(K)}{K}$ is the throughput of source i. By the model assumption, in every time slot, at most one packet is delivered in the system. Therefore,

$$\lim_{K \to \infty} \mathbb{E}\left[\sum_{i=1}^{M} \frac{N_i(K)}{K} \right] \leq C_{RA}. \tag{40}$$

Now note that by the Cauchy-Schwarz inequality, we have

$$\lim_{K \to \infty} \mathbb{E}\left[\sum_{i=1}^{M} \frac{N_i(K)}{K} \right] \mathbb{E}\left[\sum_{i=1}^{M} \frac{K}{N_i(K)} \right] \geq M^2. \tag{41}$$

Thus using (40) and (41), we find

$$\lim_{K \to \infty} \mathbb{E}\left[\sum_{i=1}^{M} \frac{K}{N_i(K)} \right] \geq \frac{M^2}{C_{RA}}. \tag{42}$$

Inserting (42) back into (39), we obtain

$$J_M^\pi \geq \frac{1}{2C_{RA}} + \frac{1}{2M}. \tag{43}$$

APPENDIX C
PROOF OF PROPOSITION 2

Suppose all packets are delivered instantaneously with one time-unit delay and without experiencing collisions. A lower bound to NAAoI in this scenario constitutes a lower bound to NAAoI in our setup. Let $X_i(m)$ denote the inter arrival time between the mth and $(m+1)$st packets. $\{X_i(m)\}_m$ is a geometric iid sequence. Under the assumption of instantaneous delivery, $I_i(m) = X_i(m)$. It hence follows from (4) that

$$\Gamma_i(m) = \sum_{k=T_i(m)}^{T_i(m)+X_i(m)-1} h_i(k) = \frac{1}{2} X_i(m)^2 + \frac{1}{2} X_i(m). \tag{44}$$

Thus, similar with [17], the time-average AoI of source i, denoted by H_i, is

$$\mathbb{E}[H_i] = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} h_i(k) = \frac{\mathbb{E}[X^2]}{2\mathbb{E}[X]} + \frac{1}{2}. \tag{45}$$

Since X in (45) has a geometric distribution with parameter θ, we find

$$\mathbb{E}[H_i] = \frac{2-\theta}{2\theta} + \frac{1}{2}. \tag{46}$$

Note that NAAoI can be captured by

$$\frac{1}{M^2} \sum_{i=1}^{M} H_i$$

and one can hence conclude that

$$J^\pi(M) \geq \frac{1}{M\theta}. \tag{47}$$

APPENDIX D
PROOF OF PROPOSITION 3

First consider a source node i whose queue is empty. This means that no new packet has arrived at that transmitter since the last delivery (from that source node) at the receiver; i.e., $h_i(k) = w_i(k)$ and hence $\delta_i(k) = 0$. Such nodes i are thus irrelevant because $\delta_j(k) \geq 0$ for all source nodes j. Now consider nodes with non-empty queues. Among these nodes, $d_i(k)$ is non-zero if and only if $(\lambda_1, \dots, \lambda_M)$ is a vector consisting of 0's except for $\lambda_i = 1$. Hence at most one $d_i(k)$ can be equal to 1. Call the corresponding source node $\ell(k)$. Expression (11) is minimized when $d_{\ell(k)}(k)$ picks the largest $\delta_j(k)$.

APPENDIX E
PROBABILITIES OF IDLE, DELIVERIES AND COLLISIONS UNDER SLOTTED ALOHA

Lemma 3: Consider any stabilized slotted ALOHA scheme. Define G as the expected number of attempted transmissions in a slot. Then, for M large, the probability of delivering a packet is (asymptotically) Ge^{-G}, the probability of idle system is (asymptotically) e^{-G}, and the probability of collisions is (asymptotically) $1 - e^{-G} - Ge^{-G}$. In particular, when $G = 1$, the maximum probability of delivery is $1/e$, the corresponding probabilities of collisions and idle system are $1 - 2/e$ and $1/e$, respectively.

Proof: The idea of the proof is very similar to [35, Chapter 4]. However, the settings are different: [35, Chapter 4] considered that packets arrive as a Poisson process (in a continuous-time system) and the buffer size is infinite, while this proof consider that packets arrive as a Bernoulli process (in a discrete-time system) and the buffer size is 1. Define the nodes that are not backlogged as *fresh* nodes. Each fresh node transmits a packet directly in a slot if it is not empty, and it generates/receives a packet with probability θ, thus a fresh node transmits a packet with probability θ. Let $P_a(i, n(k))$ be the probability that i fresh nodes transmit a packet in a time slot and let $P_s(j, n(k))$ be the probability that j backlogged nodes transmit. We have:

$$P_a(i, n(k)) = \binom{M - n(k)}{i} (1-\theta)^{M-n(k)-i} \theta^i \tag{48}$$

$$P_s(j, n(k)) = \binom{n(k)}{j} (1-p_b(k))^{n(k)-i} p_b(k)^i. \tag{49}$$

Thus, in slot k, when a packet is delivered, i.e., $\sum_{i=1}^{M} d_i(k) = 1$, the probability is

$$\Pr(\sum_{i=1}^{M} d_i(k) = 1)$$

$$= P_a\big(1, n(k)\big)P_s\big(0, n(k)\big) + P_a\big(0, n(k)\big)P_s\big(1, n(k)\big). \quad (50)$$

If the channel does not transmit a packet in a slot, i.e., we have an idle channel, $\sum_{i=1}^{M} d_i(k) = 0$, $c(k) = 0$. The probability of idle system in slot k is

$$\Pr(\sum_{i=1}^{M} d_i(k) = 0, c(k) = 0) = P_a\big(0, n(k)\big)P_s\big(0, n(k)\big). \quad (51)$$

Define the attempt rate $G = (M - n(k))\theta + n(k)p_b(k)$ as the expected number of attempted transmissions in a slot. From (48) and (49), the probability of delivery is

$$\Pr(\sum_{i=1}^{M} d_i(k) = 1)$$
$$= (M - n(k))(1 - \theta)^{M-n(k)-1}\theta(1 - p_b)^{n(k)}$$
$$+ (1 - \theta)^{M-n(k)} n(k)(1 - p_b)^{n(k)-1}p_b$$

and the probability of an idle channel is

$$\Pr(\sum_{i=1}^{M} d_i(k) = 0, c(k) = 0) = (1 - \theta)^{M-n(k)}(1 - p_b)^{n(k)}.$$

Note that the valid regime of θ is $\theta M < \frac{1}{e}$, and thus θ, p_b are small. Using the approximation $(1 - x)^{-y} \approx \exp(-xy)$ for small x, we find

$$\Pr(\sum_{i=1}^{M} d_i(k) = 1) \approx Ge^{-G}$$
$$\Pr(\sum_{i=1}^{M} d_i(k) = 0, c(k) = 0) \approx e^{-G}.$$
$$\Pr(c(k) = 1) \approx 1 - Ge^{-G} - e^{-G}.$$

Taking the first derivative of the function Ge^{-G}, we can find the maximum point is 1 for $0 < G \le 1$. So the maximum probability of delivery is

$$\Pr(\sum_{i=1}^{M} d_i(k) = 1) \approx 1/e, \quad (52)$$

correspondingly, we have

$$\Pr(\sum_{i=1}^{M} d_i(k) = 0, c(k) = 0) \approx 1/e, \quad (53)$$
$$\Pr(c(k) = 1) = 1 - 2/e. \quad (54)$$

$\square$

APPENDIX F
PROOF OF THEOREM 1

The proof is organized in three parts:

Part 1: Preliminaries. In time slot k, denote the time just before the arrival of new packets by k^- and the time just after the arrival of new packets by k^+. We hence write $\delta_i(k^-) = h_i(k^-) - w_i(k^-)$ and $\delta_i(k^+) = h_i(k^+) - w_i(k^+)$. Suppose a packet is delivered from the ith source at the end of time slot $k-1$. We then have $\delta_i\big(k^-\big) = 0$. From (14),

since all nodes have the same arrival rate and transmission policy, the sequences $\{h_i(k^-)\}_{k=1}^{\infty}$, $\{h_i(k^+)\}_{k=1}^{\infty}$, $\{w_i(k^-)\}_{k=1}^{\infty}$, $\{w_i(k^+)\}_{k=1}^{\infty}$, $\{\delta_i(k^-)\}_{k=1}^{\infty}$, $\{\delta_i(k^+)\}_{k=1}^{\infty}$ are identical random variables across $i = 1, 2, \cdots, M$, respectively. Recall that source nodes with $\delta_i(k^-) = 0$ are 0-order nodes and define $n_0(k^-)$ as the number of 0-order nodes at time k^-.

In the beginning of time slot k, on average, θM new packets arrive at the sources, and $\theta n_0(k^-)$ 0-order nodes receive new packets. Suppose source i is a 0-order node and $h_i(k^-) - w_i(k^-) = 0$. If source i receives new packets, then the source's AoI changes from $w_i(k^-)$ to $w_i(k^+) = 0$ and the destination's AoI $h_i(k^-)$ remains the same as $h_i(k^+)$. Thus,

$$\begin{aligned}\delta_i(k^+) &= h_i(k^+) - w_i(k^+) \\ &= h_i(h^+) \\ &= h_i(k^-) \\ &> h_i(k^-) - w_i(k^-) \\ &= \delta_i(k^-) \\ &= 0,\end{aligned}$$

which implies that if a 0-order source receives a new packet, then it is not a 0-order source at k^+.

Fix any large M and denote the maximum throughput of Slotted ALOHA with $C_{SA}(M)$. We know that

$$\lim_{M \to \infty} C_{SA}(M) = e^{-1}.$$

The recursion of the expected number of 0$-$order nodes is:

$$\begin{aligned}\mathbb{E}[n_0\big((k+1)^-\big)] \\ = (1 - \theta)\mathbb{E}[n_0(k^-)] + \min(M\theta, C_{SA}(M)) \quad (55)\end{aligned}$$

where the second term on the right hand side is the average number of delivered packets per time slot. Since we consider a stabilized slotted ALOHA, $\lim_{k \to \infty} \mathbb{E}[n_0(k^-)]$ exists. Denote

$$n_0^* = \lim_{k \to \infty} \mathbb{E}[n_0(k^-)].$$

Letting $k \to \infty$ on both sides of (55), we have

$$n_0^* = (1 - \theta)n_0^* + \min(M\theta, C_{SA}(M)). \quad (56)$$

Note that

$$\lim_{M \to \infty} \min(M\theta, C_{SA}(M)) = \lim_{M \to \infty} M\theta = \eta. \quad (57)$$

From (56) and (57), we have

$$\lim_{M \to \infty} \frac{n_0^*}{M} = 1. \quad (58)$$

Part 2: Find the expression of NAAoI. Using (3), we have

$$J^{SA}(M) = \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K}h_i(k^-)\right] \triangleq J_1 + J_2$$

where

$$J_1 = \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K}w_i(k^-)\right]$$

$$J_2 = \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K}\delta_i(k^-)\right].$$

Part 3: Find the limit of NAAoI. First, we consider J_1. $w_i(k^-)$ has a geometric distribution starting from 1 with parameter θ for all i. Employing the law of large number, we find

$$J_1 = \frac{1}{M\theta}. \tag{59}$$

Next, we consider J_2 and prove that its limit in large M approaches zero. Note from (12) that $\delta_i(k) = 0$ if source i is empty in time slot k and $\delta_i(k) > 0$ if a packet remains in source i in time slot k. We first note that $\delta_i(k)$ is upper bounded by $h_i(k)$. Let us consider a worse case in which buffer sizes are infinite. In this case, assuming stationarity,[9] denote the inter-arrival time and delay of packets with respect to source i by X_i and D_i. Since the Bernoulli arrival process has parameter θ, we have $\mathbb{E}[X_i] = \frac{1}{\theta} = \frac{M}{\eta}$. Moreover $\mathbb{E}[D_i]$ is approximately bounded by some constant independent of the number of sources M [55]. Now we observe that for each packet delivery the expected peak age at the destination is upper bounded by $\mathbb{E}[X_i] + \mathbb{E}[D_i]$. We can hence write

$$\mathbb{E}[\delta_i(k)|\delta_i(k) > 0] \le \mathbb{E}[X_i] + \mathbb{E}[D_i] \tag{60}$$

which implies that $\mathbb{E}[\delta_i(k)]$ is $O(M)$.

Now expand J_2:

$$
\begin{aligned}
J_2 &= \lim_{K\to\infty} \mathbb{E}[\frac{1}{M^2}\sum_{i=1}^{M}\frac{1}{K}\sum_{k=1}^{K}\delta_i(k^-)] \\
&= \lim_{K\to\infty} \mathbb{E}[\frac{1}{M^2}\frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{M}\delta_i(k^-)1_{\delta_i(k^-)>0}] \\
&= \lim_{K\to\infty} \frac{1}{M^2}\frac{1}{K}\sum_{k=1}^{K}\sum_{i=1}^{M}\mathbb{E}[\delta_i(k^-)1_{\delta_i(k^-)>0}] \\
&\le \limsup_{k\to\infty} \frac{1}{M^2}\sum_{i=1}^{M}\mathbb{E}[\delta_i(k^-)1_{\delta_i(k^-)>0}] \\
&= \limsup_{k\to\infty} \frac{1}{M^2}\sum_{i=1}^{M}\Big(\Pr(\delta_i(k^-) > 0) \\
&\qquad \times \mathbb{E}[\delta_i(k^-)|\delta_i(k^-) > 0]\Big). \tag{61}
\end{aligned}
$$

Since for k large enough the conditional expectation $\mathbb{E}[\delta_i(k^-)|\delta_i(k^-) > 0]$ is $O(M)$, it remains to prove that in the limit of large M, $\lim_{k\to\infty}\frac{1}{M}\sum_{i=1}^{M}\Pr(\delta_i(k^-) > 0)$ vanishes. But this holds because we can write

$$
\begin{aligned}
&\lim_{k\to\infty} \frac{1}{M}\sum_{i=1}^{M}\Pr(\delta_i(k^-) > 0) \\
&= \lim_{k\to\infty} \mathbb{E}[\frac{1}{M}\sum_{i=1}^{M}1_{\delta_i(k^-)>0}] \\
&= \lim_{k\to\infty} \mathbb{E}[\frac{1}{M}(M - n_0(k^-))] \\
&= \frac{M - n_0^*}{M} \tag{62}
\end{aligned}
$$

and (62) goes to zero by (58).

[9]This assumption approximately holds for infinite time horizon T

Finally, we prove that for any scheme, $J^{SA}(M)$ is lower bounded by $1/\eta$. From Proposition 2 (and letting $M \to \infty$), we have

$$\lim_{M\to\infty} J^{SA}(M) \ge \lim_{M\to\infty} \frac{1}{M\theta} = \frac{1}{\eta}.$$

Therefore, slotted ALOHA can reach the lower bound when $\theta \in (0, \frac{1}{eM}]$ and is hence optimal.

APPENDIX G
PROOF OF LEMMA 1

Before presenting the proof, we state the following straight-forward lemma (whose proof is omitted).

Lemma 4: At the beginning of time slot k, before new packets arrive at source i, $w_i(k^-) > 0$ and its probability distribution is

$$\Pr\left(w_i(k^-) = j\right) = \theta(1 - \theta)^{j-1}, \ j = 1, 2, 3, \cdots. \tag{63}$$

First consider $m = 0$ and suppose source i is a 0-order node. From Lemma 4, we know that $w_i(k^-) > 0$. Moreover, since $\delta_i(k^-) = 0$, we conclude $h_i(k^-) = w_i(k^-) > 0$. Once the 0-order node has a new arrival, $w_i(k^+) = 0$ and $h_i(k^+) = h_i(k^-)$, resulting in $\delta_i(k^+) = h_i(k^+) > 0$; i.e., the order of the node increases. In other words, the order of a 0-order node increases once it receives a new packet. In total, the fraction of 0-order nodes that become of higher order is on average $\theta\ell_0(k - 1)$. Thus,

$$\ell_0(k^+) = (1 - \theta)\ell_0(k - 1).$$

Similarly, we consider $m \ge 1$. The fraction of m-order nodes that have new arrivals is $\theta\ell_m(k - 1)$. These nodes will have larger orders. Suppose source i is of order m, $m \ge 1$, i.e., $\delta_i(k^-) = h_i(k^-) - w_i(k^-) = m$, once a new packet arrives, then $w_i(k^+) = 0$, $h_i(k^+) = h_i(k^-)$, and $\delta_i(k^+) = h_i(k^+) = m + w_i(k^-)$. From Lemma 4, $w_i(k^-) > 0$, then $\delta_i(k^+) > \delta_i(k^-) = m$. The order of a m-order node increases once it receives a new packet. In total, the fraction of m-order nodes have larger orders is $\theta\ell_m(k - 1)$.

More precisely, consider a j-order node, $j < m$. This node becomes an m-order node if it receives a new packet and $w_i(k^-) = m - j$. Using Lemma 4, we cam write

$$
\begin{aligned}
\ell_m(k^+) =&(1 - \theta)\ell_m(k - 1) \\
&+ \sum_{j=0}^{m-1}\theta\ell_j(k - 1)\Pr\left(w_i(k^-) = m - j\right) \\
=&(1 - \theta)\ell_m(k - 1) + \sum_{j=0}^{m-1}\theta\ell_j(k - 1)\theta(1-\theta)^{m-j-1} \\
=&(1 - \theta)\ell_m(k - 1) + \theta^2\sum_{j=0}^{m-1}\ell_j(k - 1)(1-\theta)^{m-j-1}. \\
& \tag{64}
\end{aligned}
$$

where the second term in (64) on the left hand side is the average fraction of nodes that have just become of order m. Denoting it by a_m, we have

$$a_m(k) = \theta^2\sum_{j=0}^{m-1}\ell_j(k - 1)(1-\theta)^{m-j-1}. \tag{65}$$

APPENDIX H
PROOF OF LEMMA 2

From the expression of ℓ_m^* in (29), $0 \leq m \leq T^* - 1$, we obtain

$$\ell_0^* = \frac{1}{eM\theta} \tag{66}$$

$$\ell_m^* = \frac{a_m^*}{\theta} \qquad 0 \leq m \leq T^* - 1. \tag{67}$$

From (26), a_m^* depends on $\{\ell_j^*\}_{j \leq m-1}$ and from (67), ℓ_m^* depends on a_m^* for $1 \leq m \leq T^*-1$. So they can be recursively found and in particular, it is not difficult to prove for all $1 \leq m \leq T^* - 1$:

$$a_m^* = \frac{\theta}{eM} \tag{68}$$

$$\ell_m^* = \frac{1}{eM}. \tag{69}$$

We prove this by mathematical induction on $T^* \geq 2$. For $T^* - 1 = 1$, the statement holds because

$$a_1^* = \theta^2 \ell_0^* = \frac{\theta}{eM}$$

$$\ell_1^* = \frac{a_1^*}{\theta} = \frac{1}{eM}.$$

Now suppose the statements (68)-(69) hold for $m \leq T^* - 1 = k$. We prove the statement for $T^* - 1 = k + 1$ and in particular we find a_{k+1}^* and ℓ_{k+1}^* below:

$$a_{k+1}^* = \theta^2 \sum_{j=0}^{k} \ell_j^* (1-\theta)^{k-j}$$

$$= \theta^2 \frac{1}{eM} \sum_{j=1}^{k} (1-\theta)^{k-j} + \theta^2 (1-\theta)^k \frac{1}{eM\theta}$$

$$= \theta^2 \frac{1}{eM} \frac{1 - (1-\theta)^k}{\theta} + \theta(1-\theta)^k \frac{1}{eM}$$

$$= \frac{\theta}{eM}. \tag{70}$$

Next, using (67), we find

$$\ell_{k+1} = \frac{1}{eM}. \tag{71}$$

Moreover, using the derivation in (70), we also find

$$a_{T^*}^* = \frac{\theta}{eM}. \tag{72}$$

Finally, from (25), we obtain

$$\ell_m^{+*} = \frac{1}{eM} \qquad 1 \leq m \leq T^* - 1. \tag{73}$$

APPENDIX I
PROOF OF THEOREM 2

Summing (29) on both sides, we have

$$\sum_{m \geq 1} a_m^* = \theta.$$

Moreover, T^* satisfies

$$T^* = \max\{t | \sum_{m \geq t} a_m^* \geq \frac{1}{eM}\} \tag{74}$$

by its definition in (19). The term $\sum_{m \geq T^*} a_m^*$ can be re-written as follows:

$$\sum_{m \geq T^*} a_m^* = \sum_{m \geq 1} a_m^* - \sum_{m < T^*} a_m^*$$

$$\overset{(a)}{=} \theta - (T^* - 1)\frac{\theta}{eM} \tag{75}$$

where (a) follows by (32) in Lemma 2. On the other hand, $\sum_{m \geq T^*} a_m^*$ satisfies the following inequality by (74):

$$\sum_{m \geq T^*} a_m^* \geq \frac{1}{eM}. \tag{76}$$

Putting (75) and (76) together, we find

$$T^* = \lfloor eM - \frac{1}{\theta} + 1 \rfloor \tag{77}$$

since T^* is an integer.

APPENDIX J
PROOF OF THEOREM 3

The proof is organized in three parts:

Part 1: Preliminaries. In this part, we discuss some notations and preliminaries which will be used in the proof. Denote the time just before arrival of new packets by k^- and the time just after arrival of new packets by k^+. Since we have assumed that all nodes are identical, the sequence $\{h_i(k^+)\}_{k=1}^{\infty}$ is identical (but not independent) across all $i = 1, 2, \cdots, M$. From (2), $\{w_i(k^+)\}_{k=1}^{\infty}$ are i.i.d with respect to i. Therefore, the sequence $\{\delta_i(k^+)\}_{k=1}^{\infty}$ is identical but not independent for all $i = 1, 2, \cdots, M$.

Since $\theta = \frac{1}{o(M)}$ and in particular $\theta > \frac{1}{eM}$, from Lemma 2, $\ell_m^{+*} = \frac{1}{eM}$ for $m = 1, 2, \cdots, T^*$ and $\ell_0^{+*} = \frac{o(M)}{eM}$. From Theorem 2, $T^* = \lfloor eM - 1/\theta + 1 \rfloor = \lfloor eM - o(M) + 1 \rfloor$. Denote

$$s_{T^*} = \sum_{m=0}^{T^*-1} \ell_m^{+*}.$$

In the limit of large M, we have

$$\lim_{M \to \infty} s_{T^*} = \lim_{M \to \infty} \frac{o(M) + \lfloor eM - o(M) + 1 \rfloor - 1}{eM} = 1. \tag{78}$$

The expected number of inactive nodes is Ms_{T^*} and the expected number of active nodes is $M(1 - s_{T^*})$.

Part 2: Find the expression of NAAoI. Let $\alpha_i = \frac{1}{M}$ for $i = 1, 2, \cdots, M$ in (3):

$$J^{SAT}(M) = \lim_{K \to \infty} \mathbb{E}[\frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \sum_{k=1}^{K} h_i(k^+)] \triangleq J_1 + J_2$$

where

$$J_1 = \lim_{K \to \infty} \mathbb{E}[\frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \sum_{k=1}^{K} w_i(k^+)]$$

$$J_2 = \lim_{K \to \infty} \mathbb{E}[\frac{1}{M^2} \sum_{i=1}^{M} \frac{1}{K} \sum_{k=1}^{K} \delta_i(k^+)].$$

In addition, $J_2 = J_{21} + J_{22}$, where

$$J_{21} = \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i:\delta_i(k^+)<\mathrm{T}^*}\delta_i(k^+)\right]$$

$$J_{22} = \lim_{K \to \infty} \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i:\delta_i(k^+)\geq\mathrm{T}^*}\delta_i(k^+)\right].$$

Part 3: Find the limit of NAAoI with respect to M. First, we consider J_1. From (2), $w_i(k^+)$ has a geometric distribution with parameter θ $\left(\text{with } w_i(k^+) = 0, 1, 2, \cdots\right)$ for all i. Let w have the same distribution as $w_i(k^+)$. We thus have

$$J_1 = \frac{1}{M}\mathbb{E}[w] = \frac{1-\theta}{M\theta}. \tag{79}$$

Next, we consider J_{21}:

$$\lim_{M\to\infty} J_{21}$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i:\delta_i(k^+)<\mathrm{T}^*}\delta_i(k^+)\right]$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i=1}^{M}\delta_i(k^+)1_{(\delta_i(k^+)<\mathrm{T}^*)}\right]$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M}\sum_{j=1}^{\mathrm{T}^*-1}\frac{\sum_{i=1}^{M}\delta_i(k^+)1_{(\delta_i(k^+)=j)}}{M}\right]$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M}\sum_{j=1}^{\mathrm{T}^*-1}\frac{j\sum_{i=1}^{M}1_{(\delta_i(k^+)=j)}}{M}\right]$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M}\sum_{j=1}^{\mathrm{T}^*-1}j\frac{\mathbb{E}[\sum_{i=1}^{M}1_{(\delta_i(k^+)=j)}]}{M}. \tag{80}$$

Substituting $\ell_j(k^+)$ for the term $\frac{\mathbb{E}[\sum_{i=1}^{M}1_{(\delta_i(k^+)=j)}]}{M}$, we find

$$\lim_{M\to\infty} J_{21} = \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M}\sum_{j=1}^{\mathrm{T}^*-1}j\ell_j(k^+). \tag{81}$$

By stationarity, note that

$$\ell_j^{*+} = \lim_{k\to\infty}\ell_j(k^+).$$

By the Cesaro Mean Lemma,

$$\lim_{K\to\infty}\frac{\sum_{k=1}^{K}\ell_j(k^+)}{K} = \ell_j^{*+}.$$

Therefore,

$$\lim_{M\to\infty} J_{21}$$

$$= \lim_{M\to\infty}\frac{1}{M}\sum_{j=1}^{\mathrm{T}^*-1}j\ell_j^{*+}$$

$$= \lim_{M\to\infty}\frac{1}{M}\frac{\mathrm{T}^*(\mathrm{T}^*-1)}{2}\frac{1}{eM} = \frac{e}{2} \tag{82}$$

where in the last step we have substituted $\ell_j^{*+} = \frac{1}{eM}$ for $j = 1, \ldots, \mathrm{T}^* - 1$ (see Lemma 2).

Finally, we consider J_{22}:

$$\lim_{M\to\infty} J_{22}$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i=1}^{M}\mathbb{E}[\delta_i(k^+)1_{\delta_i(k^+)\geq\mathrm{T}^*}]$$

$$= \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i=1}^{M}\Big(\mathbb{E}[\delta_i(k^+)|\delta_i(k^+) \geq \mathrm{T}^*]$$

$$\times \Pr(\delta_i(k^+) \geq \mathrm{T}^*)\Big)$$

$$\overset{(a)}{\leq} \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M^2}\sum_{i=1}^{M}cM\Pr(\delta_i(k^+) \geq \mathrm{T}^*). \tag{83}$$

In the above chain of inequalities, step (a) holds because $\mathbb{E}[\delta_i(k^+)|\delta_i(k^+) \geq \mathrm{T}^*] = O(M)$. To show this, we first observe that $\delta_i(k)$ is increasing in k until a delivery occurs. Now, note that $\delta_i(k^+)$ is upper bounded by T^* plus the peak age at the first delivery after time slot k. The peak age is bounded by X_i (the inter arrival time), which is $o(M)$ on average, plus delay D_i, which is constant on average (similar to (60)). The threshold T^* is also $O(M)$. So overall, we have

$$\mathbb{E}[\delta_i(k^+)|\delta_i(k^+) \geq \mathrm{T}^*] \leq cM$$

for some constant c. Note that

$$\Pr(\delta_i(k^+) = j) = \mathbb{E}[1_{\{\delta_i(k^+)=j\}}]$$

therefore

$$\frac{1}{M}\sum_{i=1}^{M}\Pr(\delta_i(k^+) \geq \mathrm{T}^*)$$

$$= \frac{1}{M}\sum_{i=1}^{M}\sum_{j\geq\mathrm{T}^*}\Pr(\delta_i(k^+) = j)$$

$$= \sum_{j\geq\mathrm{T}^*}\frac{1}{M}\sum_{i=1}^{M}\Pr(\delta_i(k^+) = j)$$

$$= \sum_{j\geq\mathrm{T}^*}\frac{1}{M}\sum_{i=1}^{M}\mathbb{E}[1_{\{\delta_i(k^+)=j\}}]$$

$$= \sum_{j\geq\mathrm{T}^*}\ell_j(k^+).$$

Again, by the Cesaro Mean Lemma,

$$\lim_{M\to\infty} J_{22}$$

$$\leq \lim_{M\to\infty}\lim_{K\to\infty}\frac{1}{K}\sum_{k=1}^{K}\frac{1}{M}cM\left(\sum_{j\geq T^*}\ell_j(k^+)\right)$$

$$= \lim_{M\to\infty}\frac{1}{M}cM\left(\sum_{j\geq T^*}\ell_j^*\right)$$

$$= \lim_{M\to\infty}\frac{1}{M}cM(1 - s_T^*)$$

$$= 0. \tag{84}$$

The last equality follows from (78) ($\lim_{M\to\infty} s_{\mathrm{T}^*} = 1$). Finally, summing J_1, J_{21} and J_{22}, we find

$$\lim_{M\to\infty}\mathbb{E}[J^{SAT}(M)] = \frac{e}{2}.$$

APPENDIX K
PROOF OF THEOREM 4

Summing (29) on both sides, we have

$$\sum_{m \geq 1} a_m^* = \theta. \tag{85}$$

From the definition of the threshold in (19), $\mathtt{T}^*$ satisfies

$$\mathtt{T}^* = \max \left\{ t \mid \sum_{m \geq t} a_m^* \geq \min \left(\theta, \frac{C^{\pi^{(1)}}}{M} \right) \right\}. \tag{86}$$

If $\theta \leq \frac{C^{\pi^{(1)}}}{M}$, we have $\mathtt{T}^* = 1$ by (85). If $\theta > \frac{C^{\pi^{(1)}}}{M}$, however, we have

$$\frac{C^{\pi^{(1)}}}{M} \leq \sum_{m \geq \mathtt{T}^*} a_m^* = \sum_{m \geq 1} a_m^* - \sum_{m < \mathtt{T}^*} a_m^*$$
$$\overset{(a)}{=} \theta - (\mathtt{T}^* - 1) \frac{\theta C^{\pi^{(1)}}}{M} \tag{87}$$

where (a) follows from (85) and (37). Using (87) and noting that $\mathtt{T}^*$ is integer, we find

$$\mathtt{T}^* = \left\lfloor \frac{M}{C^{\pi^{(1)}}} - \frac{1}{\theta} + 1 \right\rfloor. \tag{88}$$

APPENDIX L
PROOF OF THEOREM 5

The proof of Theorem 5 is almost exactly the same as that of Theorem 3. After replacing the sum arrival rate of the channel, e^{-1}, by $C^{\pi^{(1)}}$, from Part 1, Part 2 and Part 3 in the proof of Theorem 3, we have

$$J_1 = \frac{1}{M} \mathbb{E}[w] = \frac{1 - \theta}{M\theta} \tag{89}$$

$$\lim_{M \to \infty} J_{21} = \lim_{M \to \infty} \frac{1}{M} \sum_{j=1}^{\mathtt{T}^* - 1} j \ell_j^{*+}$$
$$= \lim_{M \to \infty} \frac{1}{M} \frac{\mathtt{T}^*(\mathtt{T}^* - 1)}{2} \frac{C^{\pi^{(1)}}}{M}$$
$$= \frac{1}{2C^{\pi^{(1)}}} \tag{90}$$

and

$$\lim_{M \to \infty} J_{22} \leq \lim_{M \to \infty} \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \frac{1}{M^2} \sum_{i=1}^{M} cM \Pr(\delta_i(k^+) > \mathtt{T}^*). \tag{91}$$

From Part 3 in the proof of Theorem 3, we knew that the last inequality holds because $\mathbb{E}[\delta_i(k^+) \mid \delta_i(k^+) > \mathtt{T}^*] = O(M)$. This, however, is not as oblivious here. To show this, we first observe that $\delta_i(k)$ is increasing in k until a delivery occurs. Now, note that $\delta_i(k^+)$ is upper bounded by $\mathtt{T}^*$ plus the peak age at the first delivery after time slot k. The peak age is bounded by X_i (the inter arrival time), which is $o(M)$ on average, plus delay D_i, whose expectation is upper bounded by a constant times M as formulated in Lemma 5 below.

Therefore, by the counterpart of the proof of Theorem 3, we have

$$\lim_{M \to \infty} J_{22} = 0$$

and summing J_1, J_{21} and J_{22},

$$\lim_{M \to \infty} \mathbb{E}[J^{GSAT}(M)] = \frac{1}{2C^{\pi^{(1)}}}.$$

Lemma 5: The expectation of delay, $\mathbb{E}[D_i]$, satisfies

$$\mathbb{E}[D_i] \leq c'M$$

where c' is a constant that depends on the employed transmission policy.

Proof: Recall that

$$\lim_{M \to \infty} C^{\pi^{(1)}}(M) = C^{\pi^{(1)}}.$$

Denote the inter-delivery time for source i by I_i. Thus the expected number of received packets from source i from time slot 0 to K is $\frac{K}{\mathbb{E}[I_i]}$. Since $C^{\pi^{(1)}}(M)$ is the sum throughput, we have

$$C^{\pi^{(1)}}(M) = \lim_{K \to \infty} \frac{\sum_{i=1}^{M} \frac{K}{\mathbb{E}[I_i]}}{K}.$$

Moreover, all nodes are statistically identical. Therefore,

$$C^{\pi^{(1)}}(M) = \frac{M}{\mathbb{E}[I_i]}$$

and

$$\mathbb{E}[I_i] = \frac{M}{C^{\pi^{(1)}}(M)}.$$

Note that

$$\mathbb{E}[D_i] \leq \mathbb{E}[I_i]$$

and for any $\epsilon > 0$, there exists a $N_0 > 0$ such that $C^{\pi^{(1)}}(M) \geq C^{\pi^{(1)}} - \epsilon$ for all $M \geq N_0$. Therefore,

$$\mathbb{E}[D_i] \leq \frac{M}{C^{\pi^{(1)}} - \epsilon} \triangleq c'M.$$

$\square$

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