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OPTIMAL DECISION MAKING FOR ONLINE AND OFFLINE RETAILERS UNDER BOPS MODE

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Abstract

As a new business form, the buy-online and pick-up-in-store (BOPS) mode allows consumers to pay for goods online and pick them up in a physical store. In this paper, an equilibrium model is constructed to formulate an optimal decision-making problem for online and offline retailers under the BOPS mode, where the online retailer determines the retail price of the goods and the consignment quantity in a physical store, while the offline retailer chooses the revenue share of profit by a consignment contract. Different to the existing models, the cost of overstocking and loss of understocking are incorporated into the profit function of the online retailer due to the randomness of demand. For the objective function of the offline retailer, the cross-sale quantity generated by the BOPS mode is taken into account. Then the game between the online and offline retailers is expressed as a stochastic Nash equilibrium model. Based on the analytic properties of the model, necessary conditions for the equilibrium solution are obtained. A case study and sensitivity analysis are employed to reveal the managerial implications of the model, which can provide a number of valuable suggestions on optimizing the strategies for the online and offline retailers under the BOPS mode.

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1. Introduction

Online retailing has rapidly grown in recent years all over the world, since it seems more efficient than that by brick-and-mortar retailers [8, 10]. To further amend the possible drawbacks of online selling, some online retailers have set up more than one selling channel in the competing market [6]. Thus, the optimal retail strategy, especially the coordination of online and offline retailing, has attracted research interest from the fields of applied mathematics, operation research and management sciences.

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The BOPS (buy-online and pick-up-in-store) mode is an emerging business form of online–offline integration, where consumers pay for goods online and pick them up in a physical store. For example, some famous national leading retailers in China, such as Suning and Bubugao, are exploring a few integration strategies to increase customer proposition value and cut costs [5]. These retailers allow the consumers to search and pay for the products online and pick them up in a physical store nearby. It is known that this mode has sufficiently enriched the product tangibility, reduced customers' waiting time [5] and maximized the utility of the whole supply chain [23].

As a special BOPS mode, consignment often occurs between one online retailer and another offline retailer [11]. In this case, the online retailer would like to cooperate with other physical store retailers to increase the volume of sales and reduce the stocking cost by consigning goods to the physical stores. Thus, an interesting issue in the BOPS mode is how to allocate the selling profit of the consignment goods between the online and offline retailers by a contract. For example, as an online retailer, Midea, a comprehensive enterprise of domestic appliances in China, consigns its products such as air conditioners and refrigerators to Suning supermarket, which can be regarded as an offline retailer in the BOPS mode.

Note that in real life, customers are often inclined to buy other products sold by the offline retailer as they go to pick up the goods by online purchase [12, 22]. Especially, if the consignment goods is complementary to (not a substitute for) the goods sold by the offline retailers, then extra demand can be generated from the consignment goods. Owing to existence of additional sales (cross-sales) for the offline retailer, the contract of profit allocation is necessarily involved with the selling profit of consignment goods and the cross-sale profit. Therefore, another interesting issue is to reveal what is the impact of cross-selling on the profit allocation in the BOPS mode.

With regard to a contract between the online and offline retailers, there are three types of consignment contracts in the literature: (1) price-only contracts, by which the upstream retailer charges each downstream retailer a wholesale price per unit ordered [1, 4, 15, 17]; (2) consignment price contracts, by which the upstream retailer charges the consignment price per unit unless the consignment goods is sold [1, 14]; (3) consignment contracts with revenue share, by which the downstream retailer may get the revenue share from the upstream retailers [21, 24, 26]. Particularly, for a three-echelon supply chain with a price-only contract, Seifert et al. [15] suggested that the efficiency of the upstream coordination with a random demand is greater than or equivalent to that of the downstream coordination, and the supplier and retailer would prefer to act alone instead of coordinating with the manufacturer. In push supply chains, Du et al. [4] studied the efficiency of price-only contracts, and concluded that for a lower demand uncertainty, the price-only contract is more attractive. Under a price-only contract, Sun and Debo [17] concluded that a long-term relationship between manufacturer and retailer is based on the sufficient patience of the chain members. Adida and Ratisoontorn [1] made a comparison between a consignment price contract and a consignment contract with revenue share. They concluded that the retailers gain more benefits from consignment by a consignment price contract than by a consignment contract with revenue share. Pan et al. [13] also compared the results of a consignment price contract and a consignment contract with revenue sharing. They found that a revenue-sharing contract is beneficial for the manufacturers in a two-manufacturer–one-retailer channel under the manufacturer-dominated situation. In a vendor-managed inventory (VMI) model, Shi and Xiao [16] pointed out that consumer return can decrease the retail price when the basic returns rate is substantially high; decrease the unit wholesale price when the subsidy rate of service investment is substantially high and decrease the service level but increase the subsidy rate.

Yao et al. [25] considered a revenue-sharing contract where one manufacturer and two retailers cooperate in the frame of a Stackelberg game. They showed that the supply chain can get a better performance in a revenue-sharing contract than a price-only contract. Under consignment with a revenue share, Wang et al. [21] found that the equilibrium solution between the retailer and supplier as well as their profits are affected mainly by price sensitivity and consignment cost. Chen et al. [2] dealt with coordinating a vertically separated channel with a consignment contract with revenue sharing by a Stackelberg game, and revealed that the noncooperative game leads to a lower channel profit containing a higher revenue share, a lower slotting, a higher retail price and less display space. For the risk-averse supply chain members, Xu et al. [24] found that the retail price is lower than that of a risk-neutral supply chain. They presented a two-way revenue-sharing contract such that the supply chain system can be coordinated and the members can get a win–win situation by this contract.

Though the decision-making problem in the BOPS mode has been extensively investigated in the literature, the randomness of demand and loss of understocking have not been incorporated into the existing models. Especially, under a complicated environment, one needs to quantify the impact of cross-selling on the optimal decisions and profit allocation by numerical simulation.

In this paper, to cover the in-store cost and motivate the offline retailer to cooperate in the BOPS mode, we assume that the online retailer permits the offline retailer to make his/her own revenue-sharing rate, similar to the widely used consignment contracts with revenue share (see [9, 24, 25]). By this consignment contract, the online retailer determines the online price of goods and the consignment quantity of the goods in the physical store (see [2]), while the offline retailer decides the revenue share from the selling profit generated by the consignment.

Another focus in this paper is on the quantification of the cross-sale effect. In a centralized newsvendor model, Zhang et al. [28] found that the influence of cross-selling could change the demand of goods. Hence, the order quantity and profits are affected. For the BOPS mode, we assume that the consumers can freely decide whether to purchase the consignment goods or not, after they check the goods in the physical store. Thus, the uncertainty of demand has to be taken into consideration for the online retailer. In particular, due to the uncertainty, the consignment level and the online price should be optimized by taking into account the possible cost of overstorage and loss of understocking.

Since the extra demand generated by cross-selling is closely related to the consignment level and online price, the quantity of extra demand is also uncertain for the offline retailer. Therefore, apart from the existing models available in the literature, the decision-making problem of the online retailer in this paper is formulated as a newsvendor model, and the cross-sale effect is incorporated into the objective function of the offline retailer. Consequently, the constructed model is a stochastic Nash equilibrium problem with complicated objectives: (1) the online retailer maximizes its profit by choosing optimal retail price and consignment quantity, depending on the revenue-sharing rate of the offline retailer. (2) The offline retailer maximizes its profit by selecting an optimal rate of revenue sharing from the online retailer, based on the decisions of the online retailer. Specifically, using the constructed model, we attempt to answer the following questions.

(1) How to obtain the optimal retail price, consignment quantity and the rate of revenue sharing in practice?

(2) How sensitive are the optimal decisions and the corresponding profits to the parameters of the model?

(3) What are suitable policies in practice both for the online and offline retailers?

The remainder of this paper is organized as follows. In Section 2, a stochastic Nash equilibrium model is constructed. Section 3 is devoted to the analysis on some properties of the model. A case study and sensitivity analysis are conducted in Section 4. Finally, some concluding remarks are given in Section 5.

2. Equilibrium model for online and offline retailers

In this section, we construct an equilibrium model for the online and offline retailers under the BOPS mode.

2.1. Notation and assumptions For readability, we first present the relevant notation used in this paper.

Parameters

OL: the online retailer.
OF: the offline retailer.
c_{ol}: the unit cost of ordering the consignment goods by the online retailer.
c^1_{of}: the unit cost of handling the consignment goods by the offline retailer.
c^2_{of}: the unit cost of selling the consignment goods by the offline retailer.
c_{of}: the unit cost of the offline retailer for handling and selling consignment goods.
c: the total unit cost for the double channels.
$D(p)$: the demand for the consignment goods.
$y(p)$: the expected demand at the offline retailer.
ϵ: a random scaling factor.
$f(x)$: the probability density function of ϵ.
$F(x)$: the cumulative distribution function of ϵ.
a: the primary demand.

α: the cost allocation proportion of the offline retailer in the total unit cost of double channels.
β: the price sensitivity.
L: the online retailer's expected loss.
π_{ol}: the online retailer's expected profit.
π_{of}: the offline retailer's expected profit.
v: the cross-sale factor.

Decision variables

p: the retail price.
r: the revenue sharing.
z: the stocking factor.

For a clear description of the problem, we further make the following assumptions.

(1) In the system of BOPS, there are one online retailer (OL) and one offline retailer (OF).
(2) The online and offline retailers have the same status in the game, namely, there exists neither a leader nor a follower.
(3) The online retailer first orders products from suppliers at a given cost c_{ol} and then consigns a part of the products to the offline retailer.
(4) The offline retailer has unlimited stocking capacity for the consignment quantity from the online retailer. She/he sells the goods for the online retailer to share the profit generated by the consignment sale.
(5) The unit cost of the offline retailer generated by consignment, c_{of}, is composed of two portions: the handling cost c_{of}^1 and the selling cost c_{of}^2 for each consignment goods.
(6) The consignment goods is complementary to the other goods sold in the physical store.

Denote $c = c_{\mathrm{ol}} + c_{\mathrm{of}}^1 + c_{\mathrm{of}}^2$, the total unit cost of the double channels. Then $\alpha = c_{\mathrm{of}}/c$ is the cost allocation proportion of the offline retailer in the total unit cost of double channels. Furthermore, we define $\alpha_1 = c_{\mathrm{of}}^1/c$ and $\alpha_2 = c_{\mathrm{of}}^2/c$, which represent the allocation proportions of the handling and selling costs, respectively. It is clear that $\alpha = \alpha_1 + \alpha_2 < 1$.

Suppose that the demand for the consignment goods is random and price dependent during a single selling season (see [1, 9, 14, 26]). A popular model for such a demand is specified by

$$D(p) = y(p)\epsilon, \tag{2.1}$$

where p is the retail price of the online retailer and ϵ is a random scaling factor with expectation $E[\epsilon] = 1$. Particularly, we assume that the support set of ϵ is an interval $[A, B] \subset \mathbf{R}$ $(B > A \geq 0)$ and the relation between the demand and the price is specified by

$$y(p) = ap^{-\beta}, \tag{2.2}$$

where $\beta > 1$ is called a price-sensitivity parameter and a is the primary demand (see [21, 26]). In practice, different values of β are used to reflect the potential feature of the goods. For luxury goods, β is relatively large compared to the daily necessities. Clearly, the expected demand of the consignment goods is $y(p)$.

With the above preparation, we are going to construct an equilibrium model for the optimal decision making of the online and offline retailers.

2.2. Optimization model for online retailer The online retailer will optimize the consignment quantity and retail price such that the profit is maximized in the framework of the newsvendor problem.

Let p be the retail price determined by the online retailer and let Q be the consignment quantity of the goods. In addition, by a take-it-or-leave-it consignment contract, the offline retailer gets the revenue share r from the online retailer. However, in this research, we find that the optimal revenue share $r < 1$ may be positive or negative for the offline retailer. Especially, if the profit generated by cross-sale is large enough, then the offline retailer may choose a negative r.

For a given r, the online retailer chooses the optimal retail price p and the consignment quantity Q to maximize his/her profit. Due to the uncertainty of demand, the profit objective is associated with possible cost of overstorage or loss of understocking. In particular, if the demand D is defined by (2.1) and (2.2), then the cost of overstorage is

$$L_1 = c(1-\alpha)(Q-D)^+$$

and the loss generated by the shortage of the consignment goods is referred to as

$$L_2 = (p(1-r) - c(1-\alpha))(D-Q)^+, \tag{2.3}$$

where $(\cdot)^+$ is defined by

$$(v)^+ = \begin{cases} 0, & v \le 0, \\ v, & v > 0 \end{cases}$$

and $p(1-r) - c(1-\alpha)$ is the unit opportunity loss. Thus, for the online retailer, the total cost or loss is written as

$$L = L_1 + L_2. \tag{2.4}$$

Let F and f be the cumulative distribution function and probability density function of the random parameter ϵ with a support set $[A, B]$, respectively. We call $z = Q/y(p)$ the online retailer's stocking factor and define a function $\Lambda : [A, B] \to R$ as

$$\Lambda(z) = \int_A^z (z-x) f(x)\, dx. \tag{2.5}$$

Then it is easy to prove the following results.

Proposition 2.1. *Let Λ be defined by (2.5). Then*

$$\Lambda(z) = \int_A^z F(x)\, dx.$$

PROOF. Since

$$\begin{aligned}\Lambda(z) &= \int_A^z (z-x)f(x)\,dx = \int_A^z (z-x)\,dF(x)\\ &= -(z-A)F(A) + \int_A^z F(x)\,dx\\ &= \int_A^z F(x)\,dx,\end{aligned}$$

the desired result is obtained. □

PROPOSITION 2.2. *Define*

$$l(z) = z - \Lambda(z). \tag{2.6}$$

Then l is positive and increasing when $z \geq A$.

PROOF. By Proposition 2.1,

$$l'(z) = 1 - F(z) \geq 0.$$

It indicates that l is increasing in z. As $z \geq A$, $l(z) \geq l(A) = A \geq 0$. □

Using Propositions 2.1 and 2.2, we can obtain an expression for the expectation of the total cost or loss of the online retailer.

PROPOSITION 2.3. *Let D, Λ and l be defined by (2.1), (2.5) and (2.6), respectively. Then, for the online retailer, the expectation of total cost or loss is*

$$L(p,z;r) = c(1-\alpha)y(p)\Lambda(z) + \{p(1-r) - c(1-\alpha)\}y(p)\{l(B) - l(z)\}.$$

PROOF. From the definition of D, it follows that

$$\begin{aligned}E[(Q-D)^+] &= E[y(p)(\epsilon - z)^+] = y(p)E[(\epsilon - z)^+]\\ &= y(p)\int_A^z (z-x)f(x)\,dx = y(p)\Lambda(z)\end{aligned}$$

and

$$\begin{aligned}E[(D-Q)^+] &= y(p)E[(\epsilon - z)^+]\\ &= y(p)\int_z^B (x-z)f(x)\,dx\\ &= y(p)\Big\{(x-z)F(x)\big|_z^B - \int_z^B F(x)\,dx\Big\}\\ &= y(p)\Big\{(B-z)F(B) - \int_z^B F(x)\,dx\Big\}\\ &= y(p)\Big\{B - \int_A^B F(x)\,dx - \Big(z - \int_A^z F(x)\,dx\Big)\Big\}\\ &= y(p)\{l(B) - l(z)\}.\end{aligned}$$

Thus, for the online retailer, the expectation of the total cost or loss is given by

$$L(p, z; r) = c(1-\alpha)y(p)\Lambda(z) + \{p(1-r) - c(1-\alpha)\}y(p)\{l(B) - l(z)\}.$$

Now the desired result has been proved. □

If the online retailer is risk-neutral, then an optimal decision is made by maximizing the expectation of the random profit as follows:

$$\begin{aligned} \max \quad \pi_{\rm ol}(p, z; r) &= (1-r)pE[D] - E[L] \\ &= (1-r)py(p)E[\varepsilon] - L(p, z; r) \\ &= y(p)[(1-r)p\{1 - l(B) + l(z)\} - c(1-\alpha)\{z - l(B)\}]. \end{aligned}$$

2.3. Optimization model for offline retailer Under the assumptions in this paper, the offline retailer maximizes the profit by choosing an optimal sharing rate in the total revenue generated by the consignment.

For a given decision of the online retailer (p, Q) (or (p, z)), the offline retailer attempts to maximize his/her own profit by choosing a revenue share r as large as possible. The profit of the offline retailer consists of two parts. One part is the share of the sale revenue of consignment goods from the online retailer. The other one is from the profit brought by cross-sale in virtue of the consignment goods. Denote $\pi^1_{\rm of}$ the first part of the profit. Then

$$\pi^1_{\rm of} = rp\min\{D(p), Q\} - c\alpha_1 Q - c\alpha_2 \min\{D(p), Q\}. \tag{2.7}$$

Suppose that the cross-sale quantity generated by consignment is $m(D) = kD$, where $k > 0$ is a given constant. Denote by p_0 the net profit of unit goods in the cross-sale. Then the second part of the offline retailer's profit yields

$$\pi^2_{\rm of} = p_0 m(D(p)) = kp_0 D(p). \tag{2.8}$$

Thus, in the case that the online retailer is risk-neutral, the offline retailer makes an optimal decision by maximizing the expectation of the total random profit

$$\begin{aligned} \max_r \quad \pi_{\rm of}(r; p, z) &= E[\pi^1_{\rm of} + \pi^2_{\rm of}] \\ &= rpE[\min\{D(p), Q\}] - c\alpha_1 Q - c\alpha_2 E[\min\{D(p), Q\}] + p_0 ky(p)E[\epsilon] \\ &= y(p)\{l(z)(rp - c\alpha_2) - cz\alpha_1\} + p_0 ky(p) \\ &= y(p)[rpl(z) + \{p_0 k - c\alpha_1 z - c\alpha_2 l(z)\}]. \end{aligned} \tag{2.9}$$

2.4. Nash equilibrium model between online and offline retailers Since the retailing system under the BOPS mode is involved with the interest game between the online and offline retailers, an optimal solution for this decision-making system will be determined by solving an integrated model from (2.4), (2.7) and (2.8). Specifically, we attempt to find an optimal strategy for the online and offline retailers by solving the following stochastic Nash equilibrium model:

$$\begin{cases} \max\limits_{z,p} \tilde{\pi}_{\rm ol}(z, p; r) = (1-r)pD(p) - L(p, z; r), \\ \max\limits_{r} \tilde{\pi}_{\rm of}(r; p, z) = rp\min\{D(p), Q\} - c\alpha_1 Q - c\alpha_2 \min\{D(p), Q\} + kp_0 D(p). \end{cases} \tag{2.10}$$

In general, for the stochastic problem (2.10), there does not exist any optimal solution from the viewpoint of standard optimization. However, if the online and offline retailers are assumed to be risk-neutral, then the Nash equilibrium point of the stochastic problem (2.10) can be defined as a solution of the following deterministic equilibrium model: (see [20, 27]):

$$\begin{cases} \max\limits_{z,p} \pi_{\mathrm{ol}}(z, p; r) = y(p)[(1 - r)p\{1 - l(B) + l(z)\} - c(1 - \alpha)\{z - l(B)\}], \\ \max\limits_{r} \pi_{\mathrm{of}}(r; p, z) = y(p)[rpl(z) + \{p_0 k - c\alpha_1 z - c\alpha_2 l(z)\}], \end{cases} \quad z \in [A, B]. \tag{2.11}$$

REMARK 2.4. Different from the existing models available in the literature, we formulate the decision-making problem of the online and offline retailers under the BOPS mode into a stochastic Nash equilibrium model in this paper. Especially, due to the randomness of demand, the objective functions in the model are all involved in computing complicated integrals which are associated with the decision variables. Consequently, unlike the existing results, there is no explicit analytical solution that can be employed in revealing some managerial implications from the model (see [3, 19]). In other words, a suitable numerical simulation technique should be proposed to investigate the constructed model (2.11).

3. Properties of model and solution method

In this section, we study the analytical properties of the model (2.11), which can be solved by the existing efficient algorithms for the nonlinear system of equations (see, for example, [7, 18]).

3.1. Simplification and reformulation of equilibrium model We first prove the following results.

PROPOSITION 3.1. *Let D, Λ and l be defined by (2.1), (2.5) and (2.6), respectively. Then*

$$\Lambda(B) = B - 1, \quad l(B) = 1.$$

PROOF. Since $z \in [A, B]$ and $\mu = 1$,

$$\int_A^B x f(x)\, dx = 1.$$

Define $\varphi(t) = tF(t)$. Then $\varphi'(t) = F(t) + tf(t)$ and

$$\varphi(x) = \int_A^x \{F(t) + tf(t)\}\, dt.$$

If $x = B$, then

$$\varphi(B) = \int_A^B \{F(t) + tf(t)\}\, dt = \int_A^B F(t)\, dt + \int_A^B tf(t)\, dt = \int_A^B F(t)\, dt + 1.$$

Thus,

$$\int_A^B F(t)\,dt = \varphi(B) - 1 = BF(B) - 1 = B - 1$$

and $\Lambda(z) = B - 1$. By the definitions of l and Λ, it is clear that $l(B) = B - \Lambda(B) = B - (B - 1) = 1$. □

REMARK 3.2. By Proposition 3.1, the model (2.11) can be rewritten as

$$\begin{cases} \max\limits_{z,p} \pi_{\mathrm{ol}}(z, p; r) = y(p)\{(1 - r)pl(z) + c(1 - \alpha)(1 - z)\}, \\ \max\limits_{r} \pi_{\mathrm{of}}(r; p, z) = y(p)[rpl(z) + \{p_0 k - c\alpha_1 z - c\alpha_2 l(z)\}], \end{cases} \quad z \in [A, B]. \tag{3.1}$$

3.2. Optimal decision of the online retailer Since the profit of the online retailer from selling a unit consignment of goods is $p(1 - r) - c(1 - \alpha)$, we require

$$p(1 - r) \geq c(1 - \alpha),$$

such that the online retailer is willing to choose the offline channel of consignment. The following result shows the optimality conditions for the decision making of the online retailer.

THEOREM 3.3. *For a given r, if the retail price and stocking factor (p, z) satisfy:*

(i) $z \geq 1$;
(ii) $(z - 1)/l(z) \geq (\beta - 1)/\beta$;
(iii) $(1 + \Lambda(z))/(F(z)(z - 1)) \geq (\beta - 2)/\beta$,

then the profit function of the online retailer has the unique maximizer $(p^*(r), z^*(r))$. Furthermore, $(p^*(r), z^*(r))$ is obtained by solving the following nonlinear system of equations:

$$\begin{cases} p^*(r) = \dfrac{\beta}{\beta - 1} \cdot \dfrac{c(1 - \alpha)}{1 - r} \cdot \dfrac{z^* - 1}{l(z^*)}, \\ 1 - F(z^*) = \dfrac{c(1 - \alpha)}{p^*(r)(1 - r)} = \dfrac{\beta - 1}{\beta} \cdot \dfrac{l(z^*)}{z^* - 1}. \end{cases} \tag{3.2}$$

PROOF. We first prove the first equation in (3.2). From the definitions of $y(p)$ and $\pi_{\mathrm{ol}}(p, z; r)$, it follows that for a given r,

$$\begin{aligned} \frac{dy(p)}{dp} &= -\frac{\beta}{p}(ap^{-\beta}) = -\frac{\beta}{p} y(p), \\ \frac{\partial \pi_{\mathrm{ol}}(p, z; r)}{\partial p} &= -\frac{\beta}{p} y(p)[p(1 - r)l(z) + c(1 - \alpha)(1 - z)] + y(p)(1 - r)l(z) \\ &= y(p)\Big[(1 - \beta)(1 - r)l(z) - \frac{\beta}{p} c(1 - \alpha)(1 - z)\Big]. \end{aligned}$$

Since $y(p) > 0$,

$$(1 - \beta)(1 - r)l(z) - \frac{\beta}{p} c(1 - \alpha)(1 - z) = 0.$$

Thus,

$$p = \frac{\beta}{\beta - 1} \cdot \frac{c(1-\alpha)}{1-r} \cdot \frac{z-1}{l(z)} \equiv p^*(r).$$

From the conditions (i), (ii) and $p^*(r)(1-r) \geq c(1-\alpha)$, it is clear that $p^*(r) \geq 0$.

Next, we prove that the second equation in (3.2) holds. For a given r, since $y(p) > 0$ and

$$\frac{\partial \pi_{\mathrm{ol}}(p, z; r)}{\partial z} = y(p)[p(1-r)(1-F(z)) - c(1-\alpha)] = 0,$$

$$p(1-r)(1-F(z)) - c(1-\alpha) = 0.$$

Thus,

$$1 - F(z) = \frac{c(1-\alpha)}{p(1-r)}.$$

Based on the first equation in (3.2),

$$1 - F(z^*) = \frac{c(1-\alpha)}{p^*(r)(1-r)} = \frac{\beta - 1}{\beta} \cdot \frac{l(z^*)}{z^* - 1}.$$

Finally, we prove that the profit function of the online retailer has a unique maximizer. Taking derivatives with respect to z on both sides of the equation

$$1 - F(z) = \frac{\beta - 1}{\beta} \cdot \frac{l(z)}{z - 1}, \tag{3.3}$$

$$-f(z) = \frac{\beta - 1}{\beta} \cdot \frac{z - zF(z) - 1 + F(z) - z - \Lambda(z)}{(z-1)^2}.$$

Thus,

$$f(z) = \frac{\beta - 1}{\beta} \cdot \frac{F(z)(z-1) + 1 + \Lambda(z)}{(z-1)^2}.$$

From condition (i), it is clear that $f(z) \geq 0$. On the other hand,

$$\begin{aligned}
\frac{d^2 \pi_{\mathrm{ol}}(p, z; r)}{dz^2} &= \frac{d}{dz}\Big(\frac{d\pi_{\mathrm{ol}}(p, z; r)}{dz}\Big) = \frac{d}{dz}\Big(\frac{\partial \pi}{\partial p} \cdot \frac{dp}{dz} + \frac{\partial \pi}{\partial z}\Big) \\
&= \frac{d}{dz}\Big(\frac{\partial \pi}{\partial p} \cdot \frac{\beta}{\beta - 1} \cdot \frac{c(1-\alpha)}{1-r} \cdot \frac{F(z)(z-1) - \{1 + \Lambda(z)\}}{l^2(z)} + \frac{\partial \pi}{\partial z}\Big) \\
&= \beta \frac{c(1-\alpha)}{1-r} \cdot \frac{F(z)(z-1) - \{1 + \Lambda(z)\}}{l^2(z)} \cdot y(p)\{1 - F(z)\} - y(p) p f(z).
\end{aligned}$$

Denote

$$G(z) = \beta \frac{c(1-\alpha)}{1-r} \cdot \frac{F(z)(z-1) - \{1 + \Lambda(z)\}}{l^2(z)} \cdot y(p)\{1 - F(z)\}.$$

Then equation (3.3) yields

$$G(z) = (\beta - 1) \frac{c(1-\alpha)}{1-r} y(p) \frac{F(z)(z-1) - \{1 + \Lambda(z)\}}{(z-1) l(z)}$$

and, therefore,

$$\begin{aligned}\frac{d^2\pi_{\mathrm{ol}}(p,z;r)}{dz^2} &= G(z) - y(p)pf(z)\\ &= \frac{c(1-\alpha)}{1-r}\cdot\frac{y(p)}{(z-1)l(z)}\cdot[(\beta-2)F(z)(z-1)-\beta\{1+\Lambda(z)\}].\end{aligned}$$

From condition (iii), it is clear that

$$\frac{d^2\pi_{\mathrm{ol}}(p,z;r)}{dz^2} \le 0.$$

Therefore, $(p^*(r), z^*)$ is the unique maximizer of the profit function $\pi_{\mathrm{ol}}(p,z;r)$. □

3.3. Optimal share of offline retailer For any given decision of the online retailer, the offline retailer will choose an optimal share r from the total revenue to maximize his/her own expected profit $\pi_{\mathrm{of}}(r;z,p)$. We first prove the following result.

PROPOSITION 3.4. *If z, p and r satisfy (3.2), then*

$$\frac{dp}{dr} = \frac{p}{1-r}.$$

PROOF. Since

$$\frac{dp(r)}{dr} = \frac{\beta}{\beta-1}\cdot\frac{z-1}{l(z)}\cdot c(1-\alpha)\cdot\Big(\frac{1}{1-r}\Big)' = \frac{p}{1-r},$$

the proof of the proposition is complete. □

Based on Proposition 3.4, we obtain an optimality condition for the revenue share rate.

THEOREM 3.5. *For a given (p,z), the optimal share rate of the offline retailer, $r^*(p,z)$, satisfies*

$$r^*(p,z) = \frac{(\beta-1)X(z) - c(1-\alpha)(z-1)}{(\beta-1)X(z) - \beta c(1-\alpha)(z-1)}, \tag{3.4}$$

where $X(z) = p_0k - c\alpha_1 z - c\alpha_2 l(z)$. Furthermore, the optimal share rate is unique for a given pair (p,z).

PROOF. From (2.9),

$$\begin{aligned}\frac{d\pi_{\mathrm{of}}(r;z,p)}{dr} &= -\frac{\beta}{1-r}y(p)\{rpl(z)+p_0k-c\alpha_1 z-c\alpha_2 l(z)\} + y(p)\Big\{pl(z)+rl(z)\frac{p}{1-r}\Big\}\\ &= \frac{y(p)}{1-r}\{(1-r\beta)pl(z)-\beta X(z)\}.\end{aligned}$$

As an optimal share rate of the offline retailer, $r^*(p,z)$ necessarily satisfies

$$\frac{d\pi_{\mathrm{of}}(r;z,p)}{dr} = 0,$$

so, by Theorem 3.3, we obtain the desired result (3.4).

Next, we prove the uniqueness of $r^*(p,z)$. Since

$$\begin{aligned}\frac{d^2\pi_{\mathrm{of}}(r;z,p)}{dr^2} &= l(z)\frac{y(p)}{1-r}\Big\{\frac{p}{1-r}(1-r\beta)+p(-\beta)\Big\}\\ &= l(z)\frac{y(p)}{1-r}\Big\{\Big(\frac{1-r\beta}{1-r}-\beta\Big)\cdot p\Big\}\\ &= \frac{y(p)}{1-r}l(z)p\frac{1-\beta}{1-r}<0,\end{aligned}$$

we conclude that for any given pair (p,z), $r^*(p,z)$ is the unique maximizer of the objective function π_{of}. This implies that the optimal share rate is unique for the given pair (p,z). □

Based on Theorems 3.3 and 3.5, we present optimality conditions for the equilibrium solution of the model (3.1).

Theorem 3.6. *Let (p^*,z^*,r^*) be the equilibrium solution of the model (3.1) or (2.11). Assume that the conditions in Theorem 3.3 are satisfied. Then (p^*,z^*,r^*) solves the following nonlinear system of equations:*

$$\begin{cases} p=\dfrac{\beta}{\beta-1}\cdot\dfrac{c(1-\alpha)}{1-r}\cdot\dfrac{z-1}{l(z)},\\ 1-F(z)=\dfrac{\beta-1}{\beta}\cdot\dfrac{l(z)}{z-1},\\ r=\dfrac{(\beta-1)X(z)-c(1-\alpha)(z-1)}{(\beta-1)X(z)-\beta c(1-\alpha)(z-1)},\end{cases} \tag{3.5}$$

where $X(z)=p_0k-c\alpha_1z-c\alpha_2l(z)$.

Remark 3.7. By Theorem 3.6, the model (3.1) can be converted into the nonlinear system of equations (3.5) in order to find the equilibrium solution. Thus, the existing efficient algorithms for the nonlinear system of equations can be used to solve this model. In this paper, we directly apply the modified nonmonotone Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm to solve (3.5), which was developed by Wan et al. [18] very recently.

4. Case study and sensitivity analysis

In this section, we investigate some different scenarios of the constructed model. Sensitivity analysis is employed to reveal some underlying managerial implications.

According to the assumption on the randomness of demand, we first construct a class of probability density functions for the random variable ϵ in (2.1). For simplicity, we use the following cubic function as the model of the density function:

$$f(x)=k_0x(x-b_1)(x-b_2),$$

where k_0, b_1 and b_2 are appropriately chosen constants. Specifically, if $A = 0$ and $B = 2$ in (2.1), then the density function f satisfies

$$\begin{cases} \int_0^2 f(x)\,dx = F(2) = 1, \\ \int_0^2 xf(x)\,dx = E[\epsilon] = 1. \end{cases}$$

It yields

$$\begin{cases} k_0(4 - \frac{8}{3}(b_1 + b_2) + 2b_1 b_2) = 1, \\ k_0(\frac{32}{5} - 4(b_1 + b_2) + \frac{8}{3}b_1 b_2) = 1. \end{cases} \tag{4.1}$$

From (4.1), we obtain a class of density functions as follows:

$$f(x) = \frac{b_1 - 2}{-(20/3)b_1^2 + (56/5)b_1 + 8/5} \cdot x(x - b_1)\Big(x - \frac{2b_1 - (18/5)}{b_1 - 2}\Big).$$

Especially, if we take $b_1 = 3$, then the density function of ϵ is

$$f(x) = \frac{x}{4}\Big(x - \frac{12}{5}\Big)(x - 3).$$

To show the effect of the cross-sale on the decision making, we call $v = p_0 k$ the cross-sale factor. It is clear that v represents the profit from the unit cross-sale generated by the unit consignment goods. Using (2.3) and (3.2), we rewrite the opportunity loss as

$$\begin{aligned} s &= p(1 - r) - c(1 - \alpha) \\ &= c(1 - \alpha)\Big\{\frac{\beta}{\beta - 1} \cdot \frac{z - 1}{l(z)} - 1\Big\}. \end{aligned} \tag{4.2}$$

From equation (4.2), it is clear that the opportunity loss is affected by two parameters β and α, the price sensitivity and the cost-allocation proportion, respectively, for the given total unit cost c. Thus, the effects of β and α can reflect the opportunity loss. We will focus on sensitivity analysis on the parameters: the price sensitivity and the cost-allocation proportion of the offline retailer. In addition, we take the parameters of the model as

$$a = 10, \quad c = 10, \quad \alpha_1 = 0.6\alpha, \quad \alpha_2 = 0.4\alpha.$$

4.1. Effect of price sensitivity We first study the impact of price sensitivity on the equilibrium solution and the allocation of profits.

We change the values of β, the price sensitivity, from 1.6 to 2.8 with a step size 0.01. Additionally, we take three different cross-sale factors $v_1 = 8$, $v_2 = 8.5$ and $v_3 = 9$ to show the impact of the cross-sales.

In Figure 1, we plot the dependence of the optimal revenue sharing r^* on the price sensitivity under the three different cross-sale factors.

From Figure 1, we concluded the following.

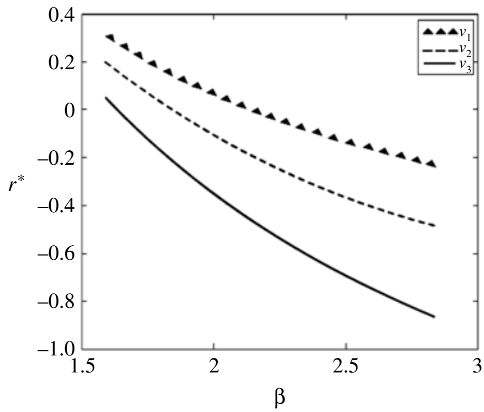

FIGURE 1. Optimal revenue sharing r^*.

(1) For the given cross-sale factors, the offline retailer would like to get a negative revenue share from the online retailer when the price sensitivity is large enough. In other words, the offline retailer prefers to share the profit from the cross-sale with the online retailer as the price sensitivity is large enough.
(2) The share rate of profit decreases with the price sensitivity. That means, for the consignment goods with a price sensitivity large enough, the offline retailer prefers to share a greater proportion of the cross-sale profit with the online retailer.
(3) The sharing rate of profit also depends on the cross-sale factor; the larger the cross-sale factor, the smaller the revenue share. In other words, the offline retailer does not prefer to share the cross-sale profit with the online retailer while the cross-sale factor is large.
(4) If the online retailer hopes to share more cross-sale profit, he/she may consign some products with higher price sensitivity.

In Figure 2, we plot the curve which describes the dependence relation between the optimal price and price sensitivity. Figure 2 indicates the following.

(1) The online retailer's optimal retail price decreases as the price sensitivity β becomes larger. It is in accordance with the practical situation; if the consumers are sensitive to the price of goods, the online retailer may obtain more profit by reducing the retail price.
(2) The optimal retail price is greatly affected by the cross-sale factor; the optimal price is decreasing with respect to the cross-sale factor. In other words, for a smaller cross-sale factor, the online retailer would like to choose a higher retail price.

In Figure 3, the relation between the optimal stocking factor and price sensitivity for different cross-sale factors is presented. The following is shown in Figure 3.

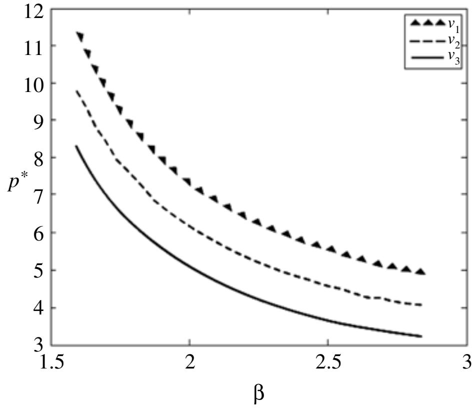

FIGURE 2. Optimal retail price p^*.

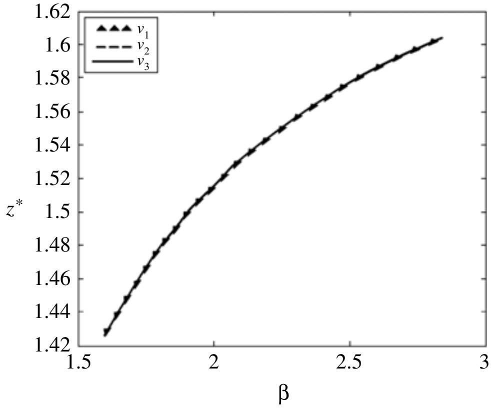

FIGURE 3. Optimal stocking factor z^*.

(1) The online stocking factor is increasing in the price sensitivity. As the price sensitivity becomes larger, the online retailer has a stronger desire to order the consignment goods.
(2) The cross-sale factor seems to generate less influence on the optimal stocking factor. Actually, from the condition

$$1 - F(z^*) = \frac{\beta - 1}{\beta} \cdot \frac{l(z^*)}{z^* - 1},$$

it is easy to see that the cross-sale factor does not affect the stocking factor.

Finally, we are interested in the allocation of the profits with different price sensitivity and cross-sale factors. In Figure 4, we plot the profit allocation between the online and offline retailers as the price sensitivity increases. It is clear that:

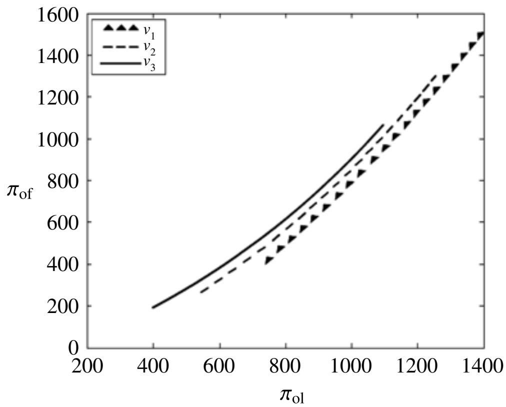

FIGURE 4. Maximal expected profits π_{ol} and π_{of}.

(1) as the price sensitivity increases, the expected profits of the online and offline retailers decrease. It implies that the consignment goods with higher price sensitivity can bring less profit for both the online and offline retailers;
(2) the expected profits of the online and offline retailers both increase as the cross-sale factor v becomes smaller;
(3) lower cross-sale factor is helpful to increase the profits of the online and offline retailers.

4.2. Effect of double-channel cost allocation Similar to the analysis on the effect of price sensitivity, we now evaluate the effects of the double-channel cost-allocation proportion on the equilibrium solutions and allocation of profits. In addition, to show the impact of cross-sales, we also conduct the analysis under three different cross-sale factors: $v_1 = 8$, $v_2 = 8.5$ and $v_3 = 9$.

We change the values of α, the double-channel cost-allocation proportion of the offline retailer, from 0.05 to 0.95 with a step size 0.01. The price sensitivity is fixed by $\beta = 3$. In Figure 5, we plot the relation curve between the optimal rate of revenue share in the consignment contract and the double-channel cost-allocation proportion.

From Figure 5, it follows that:

(1) for different proportions of double-channel cost allocation, the optimal rate of revenue share increases with respect to the cost-allocation proportion of the offline retailer. For a smaller cost-allocation proportion, the revenue share rate is negative, which means that if the offline retailer bears less double-channel cost, she/he will share more double-channel profits from the cross-sale with the online retailer. Only in the case that the cost-allocation proportion of the offline retailer is larger enough does the share rate of double-channel revenue become positive;
(2) for the offline retailer, a higher cross-sale factor generates a smaller sharing rate of the cross-sale profit.

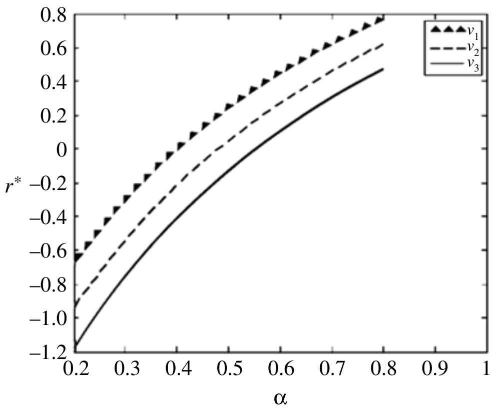

FIGURE 5. Optimal revenue sharing r^*.

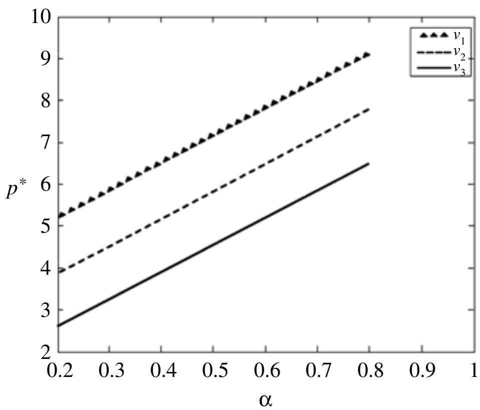

FIGURE 6. Optimal retail price p^*.

In Figure 6, the relation between the optimal retail price and cost-allocation proportion of the offline retailer is presented. Figure 6 demonstrates that:

(1) the optimal retail price increases if the offline retailer would like to accept a greater cost-allocation proportion of the offline retailer. The reason lies in that the online retailer has to give a positive proportion of the profit generated by consignment to the offline retailer as the cost-allocation proportion of the offline retailer increases. Thus, in order to guarantee his/her own profit, the offline retailer has to raise the retail price;

(2) for a larger cross-sale factor, the optimal retail price is lower for the online retailer. Actually, an increasing cost-allocation proportion of the offline retailer leads to an increasing revenue share, and the optimal retail price also goes up.

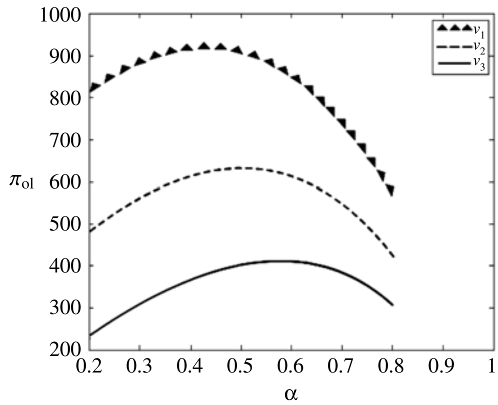

FIGURE 7. Expected profits of the online retailer.

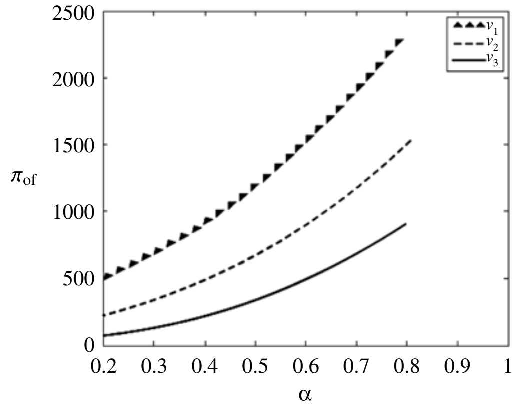

FIGURE 8. Expected profits of the offline retailer.

Similarly, because the optimal stocking factor satisfies

$$1 - F(z^*) = \frac{\beta - 1}{\beta} \cdot \frac{l(z^*)}{z^* - 1}, \tag{4.3}$$

we conclude that z^* does not depend on the cross-sale factor or the cost-allocation proportion of the offline retailer. Actually, from (4.3), it follows that the optimal stocking factor is only related with the price sensitivity β.

Finally, we show the impact of the cost-allocation proportion of the offline retailer and cross-sale factor on the profits of the online and offline retailers in Figures 7 and 8, respectively. We observe the following.

(1) As the double-channel cost-allocation proportion of the offline retailer increases, the profit of the online retailer increases in the case that the double-channel cost-allocation proportion is less than a critical point. For a proportion of the

double-channel cost allocation which is greater than the critical point, the profit decreases in the cost-allocation proportion of the offline retailer.

(2) As for the offline retailer, the expected profit increases with respect to the double-channel cost-allocation proportion of the offline retailer. It is in accordance with the practical situation.

(3) The expected profits of both the online and offline retailers increase with the cross-sale factor. Lower cross-sale factor is helpful to increase the profit of the online and offline retailers.

5. Conclusions

In this paper, we have constructed a stochastic Nash equilibrium model for the decision-making problem of the online and offline retailers under the BOPS mode. Compared to the existing models in the literature, we consider the cost of overstocking, the loss of understocking and the cross-sale revenue generated by consignment in the objective functions of the model.

Based on the property analysis on our model, the optimality conditions have been presented for the equilibrium solution of the model, which are useful to find the numerical solution of the model by the existing efficient algorithms for the nonlinear system of equations.

A case study and sensitivity analysis have revealed the following valuable managerial implications of the model.

(1) Price sensitivity, cost-allocation proportion and cross-sale factor have a substantial influence on the optimal decisions of the offline and online retailers. For example, the online retailer should choose his/her optimal stocking factor according to the price sensitivity of the consignment goods. As the price sensitivity increases, the optimal stocking factor also goes up. In this case, the expected profits of the online and offline retailers are decreasing, that is, the consignment goods with higher price sensitivity can bring less profit for both the online and offline retailers. However, the cross-sale factor does not affect the optimal stocking factor.

(2) For the offline retailer, if the consignment goods can only generate a lower cross-sale, then his/her optimal revenue share from the online retailer is greater, and the optimal retail price of the online retailer also becomes higher.

(3) For the online retailer, his/her maximal expected profit is significantly affected by the double-channel cost-allocation proportion of the offline retailer. For a proportion of the offline retailer less than a critical point, the profit of the online retailer increases with respect to the double-channel cost-allocation proportion. However, for a double-channel cost-allocation proportion greater than the critical point, the profit of the online retailer decreases as the double-channel cost-allocation proportion increases.

(4) The expected profits of both the online and offline retailers decrease with the cross-sale factor. A lower cross-sale factor is helpful to give rise to larger expected profits for the online and offline retailers.

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